SymmetryScapes West:
An Exhibition of Mathematical Art

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Why symmetry? Symmetry seems to be a universal human experience. It is encoded in the rhythms of our bodies and the cycles of the seasons. In visual art, symmetry is a sort of comfort food for our eyes. The surprise is that there are mathematical limitations on symmetry types, which means that every human who has ever attempted to make perfectly repeating patterns has ended up making patterns that are, in some sense, the same as the ones you see here.

You can learn about the mathematics behind the patterns in this exhibition from my book, *Creating Symmetry, The Artful Mathematics of Wallpaper Patterns*. But first, I invite you simply to enjoy the experience of symmetry in the patterns offered here.

Images of Symmetry

We perceive an enjoyable sameness in a pattern when some transformation leaves it unchanged. Such a transformation is called a *symmetry* of the pattern. We begin with patterns in the plane, the familiar plane of Euclidean geometry. Candidates for symmetries of a pattern are the rigid motions of the plane such as translations and rotations. (A visual glossary explains these terms.)

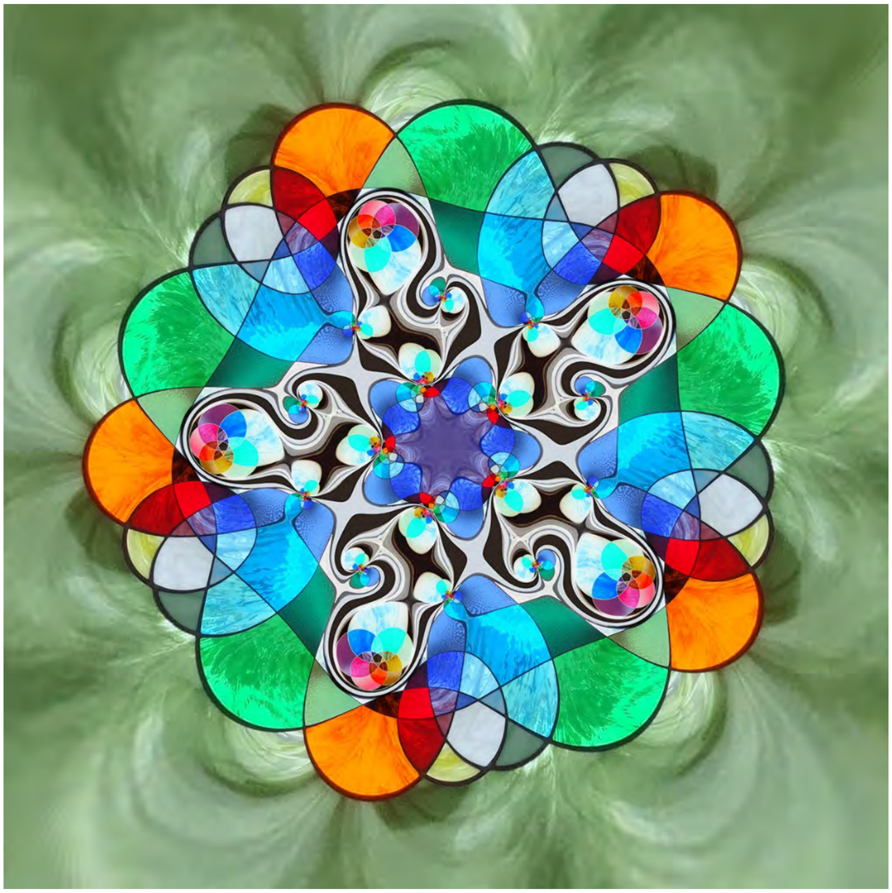
A celebrated classification result tells us that patterns fall into three types: wallpapers, friezes, and rosettes, according to how much translational symmetry the pattern enjoys. A wallpaper pattern has translational symmetry in two directions, a frieze pattern in only one. Rosettes have no translational symmetry at all, and that is a natural place to begin.

Glass Mandalas, Red and Green

digital prints on aluminum, 2016, two at 20” by 20” and one 8” by 8”

Mandalas invite the eye to wander in circular or spiral patterns, finding places where the design repeats itself. In each mandala, we can find the panes of the stained glass window that gives its colors to the design. Each mandala was created using a mathematical formula to reference pixels in the source photograph of the window, as well as its negative. Both mandalas have five-fold rotational symmetry, an uncommon choice in mandala design, but one that echoes the symmetry of the window, which was made by Hans Schepker from my design.

8 by 8 should hang between two mandalas

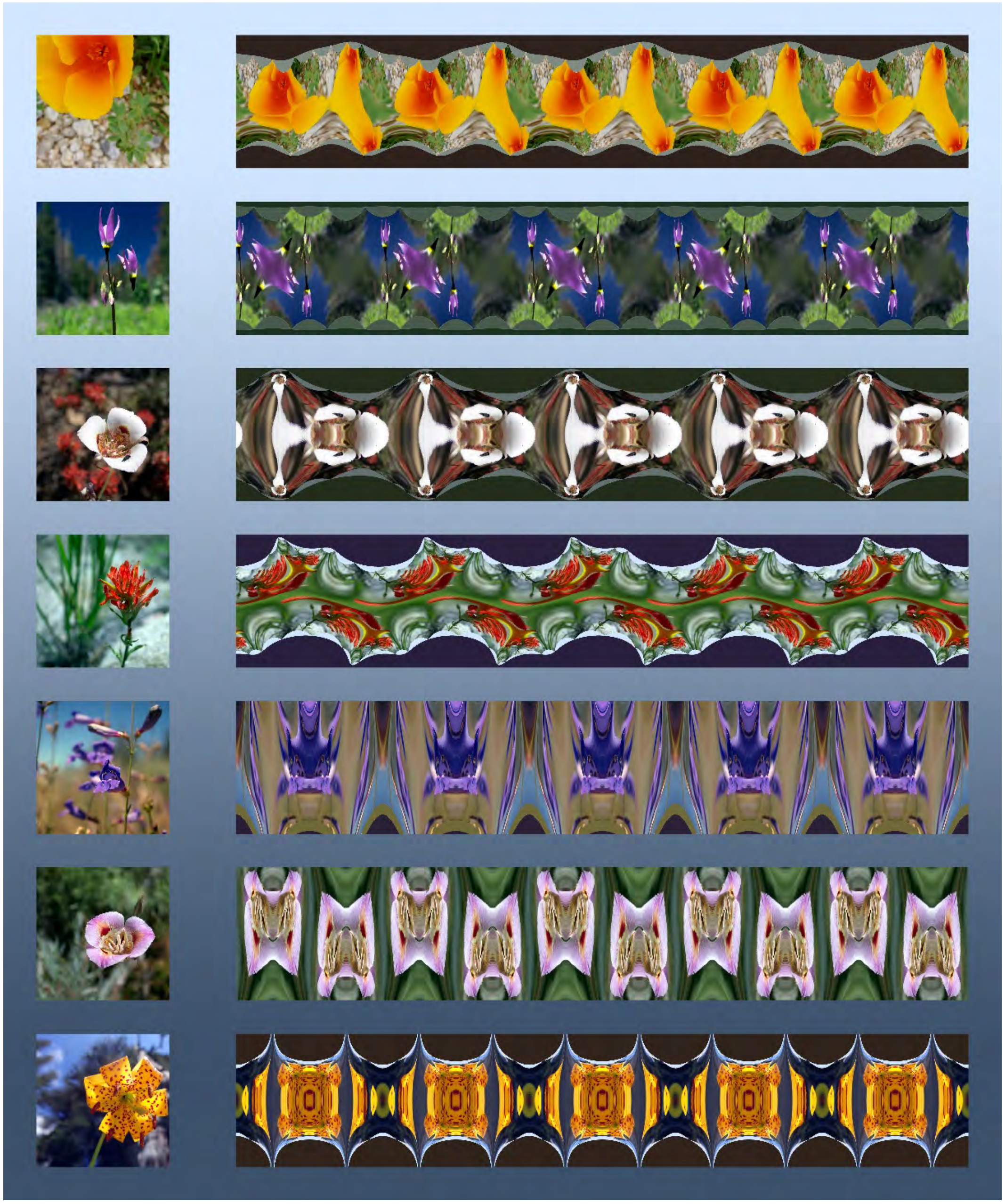


A Frieze Sampler

digital print on aluminum, 2016, 20” by 24”

Friezes take their name from an architectural feature: a band of decoration typically along the top of a wall. In the theory of plane symmetry, the first famous classification result says that there exist exactly seven types of frieze patterns. Every pattern ever constructed by anyone, as long as it repeats exactly along one direction, can be classified as belonging to one of these seven types.

Mathematicians, like many humans, like to collect exactly one artifact of each type. Here is my sampler of seven friezes, made from source photographs of California wildflowers. I especially like the fourth one down, which exemplifies a *glide reflection symmetry*—a flip-and-slide motion that leaves the pattern unchanged.



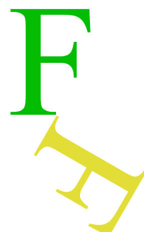
A Visual Glossary of Isometries

An *isometry* is a transformation of the plane that leaves distances unchanged.

Translation

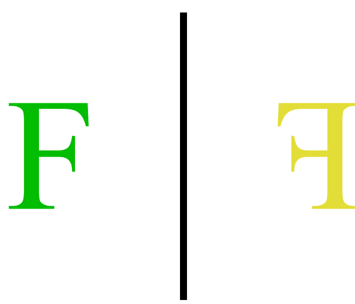
A *translation* slides the plane along itself.

Rotation



A *rotation* turns the plane about a fixed point. This rotation turns through 120 degrees. When you do that three times, you return to the start. We call this *3-fold rotation*.

Reflection



A *reflection* flips the plane about a line, called the *mirror axis*. Reflections are called *indirect* isometries, because they turn shapes over and make writing appear backward.

Glide Reflection

A *glide reflection* flips the plane about a line, and slides it along the *same* line.

Pinecones from White Bark Vista

digital print on aluminum, 2013, 20” by 20”

Part of the enjoyment of wallpaper patterns is the way your mind knows how to continue the pattern outside the given frame, in both the left/right and up/down directions. (When mathematicians use the word *wallpaper*, we just mean any pattern with translational symmetry in two independent directions.)

The undulating flip-and-slide symmetry in the up/down direction makes this pattern type one of my favorites. It is symmetric, but not overly so. Here we see alternating axes of glide reflections that are not related by translation. This pattern type is called pg by the International Union of Crystallographers.

Carousel Horses from Pumpkin Pie

digital print on aluminum, 2015, 20” by 20”

This pattern enjoys glide symmetry, with one row of horses facing left and the one below facing right. Unlike the previous pattern, this one has rotational symmetry−you can spin it 180° about the horse centers and the entire pattern remains invariant.

A regret: The constraints of the mathematical process kept me from getting the prettiest colors of the pansies into the image, unless I gave up the horse shapes. The process of selecting an image from infinite possibilities is highly experimental; the mathematical formulas are both extremely flexible and hard to control exactly.

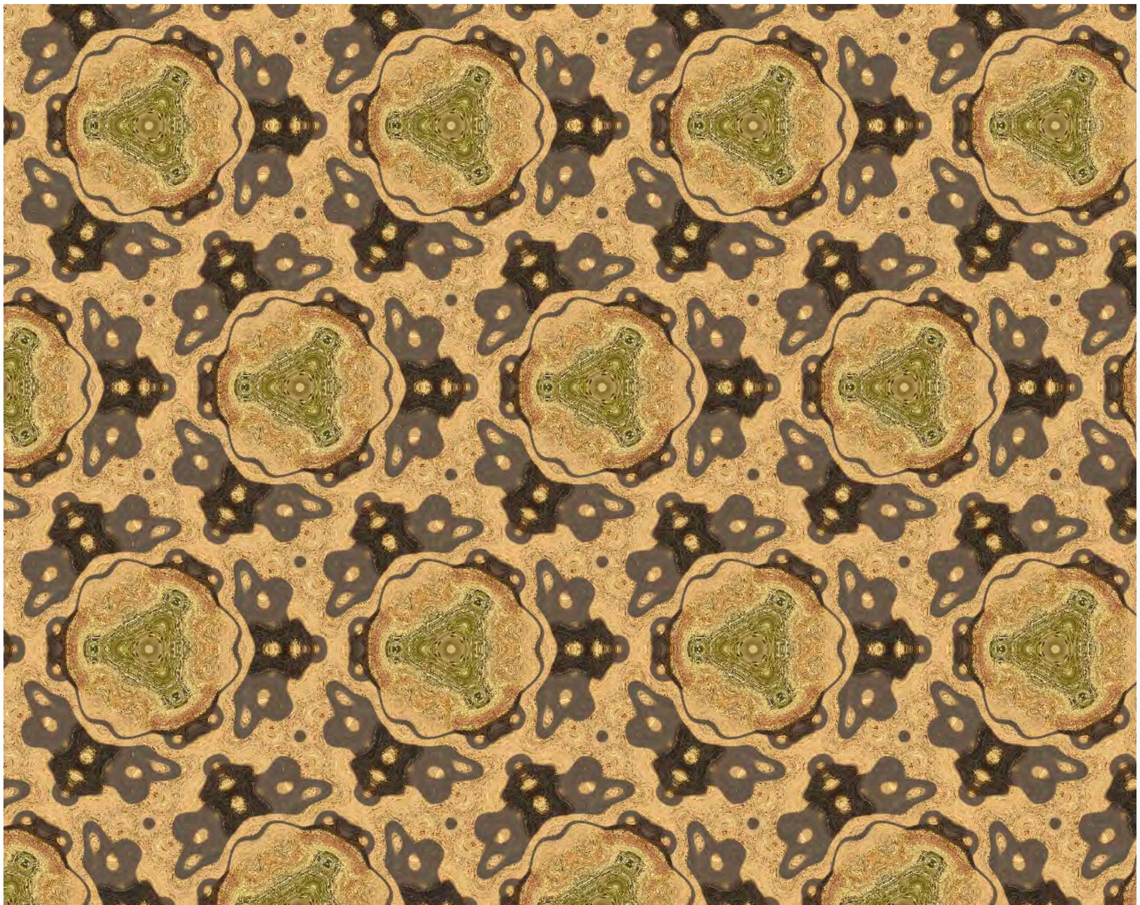
Autumn Moths from a Hike in the Hills

digital print on aluminum, 2016, 20” by 20”

The big surprise about wallpaper patterns is that there are exactly 17 types. After making many examples of each type, I still return to the type of this pattern as a favorite; it’s called p31m.

Notice the three-fold rotational symmetry, but also the alternation of mirror symmetry with three-way pinwheels. For me, this gives a beautiful balance of sameness and variation. It’s symmetric, yet organic.

I took the source photograph on an autumn day, hiking in the golden hills above San Jose, CA. This piece shows that interesting patterns can arise from photographs with rather limited color palettes.



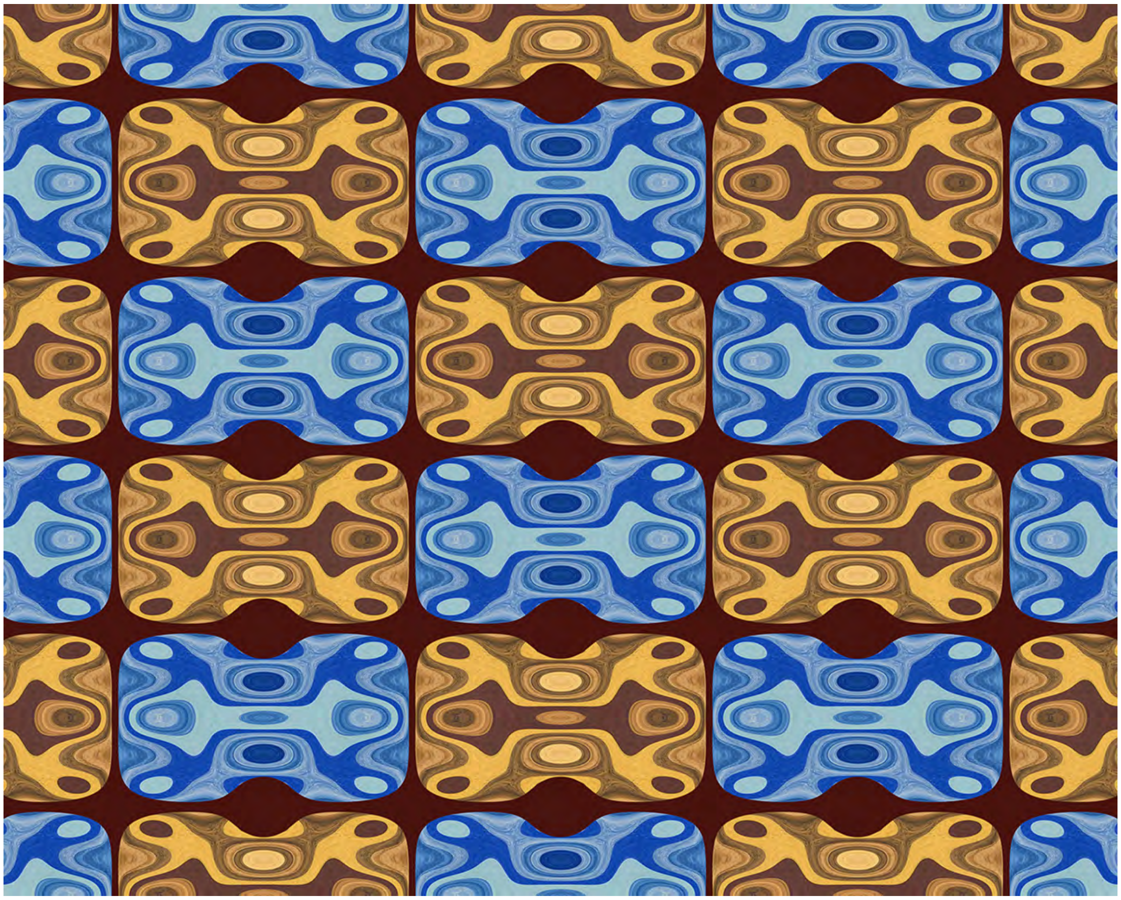
Color-Reversing Lizards from a Wooden Coaster

digital print on aluminum, 2014, 20” by 16”

The next step beyond repeating the same thing again and again is to repeat the thing in a different color. Humans have done this for millennia. Given a collection of mathematical formulas that produce repeats, it’s easy to turn them into formulas that produce repeats in altered colors.

In this image, you can flip the pattern about various horizontal and vertical lines and *every color in the whole pattern* will swap to its opposite. This is why we call it a *color-reversing* pattern.

What is an opposite color? For those old enough to have seen color film negatives, that’s what I mean. In a digital world, we can just say that there’s a well-defined way to take the negative of any color, which reverses hues and turns light colors to dark ones. Here, the rich browns of the woods become equally rich blues. The maroon color is considered neutral, as if it were its own negative.

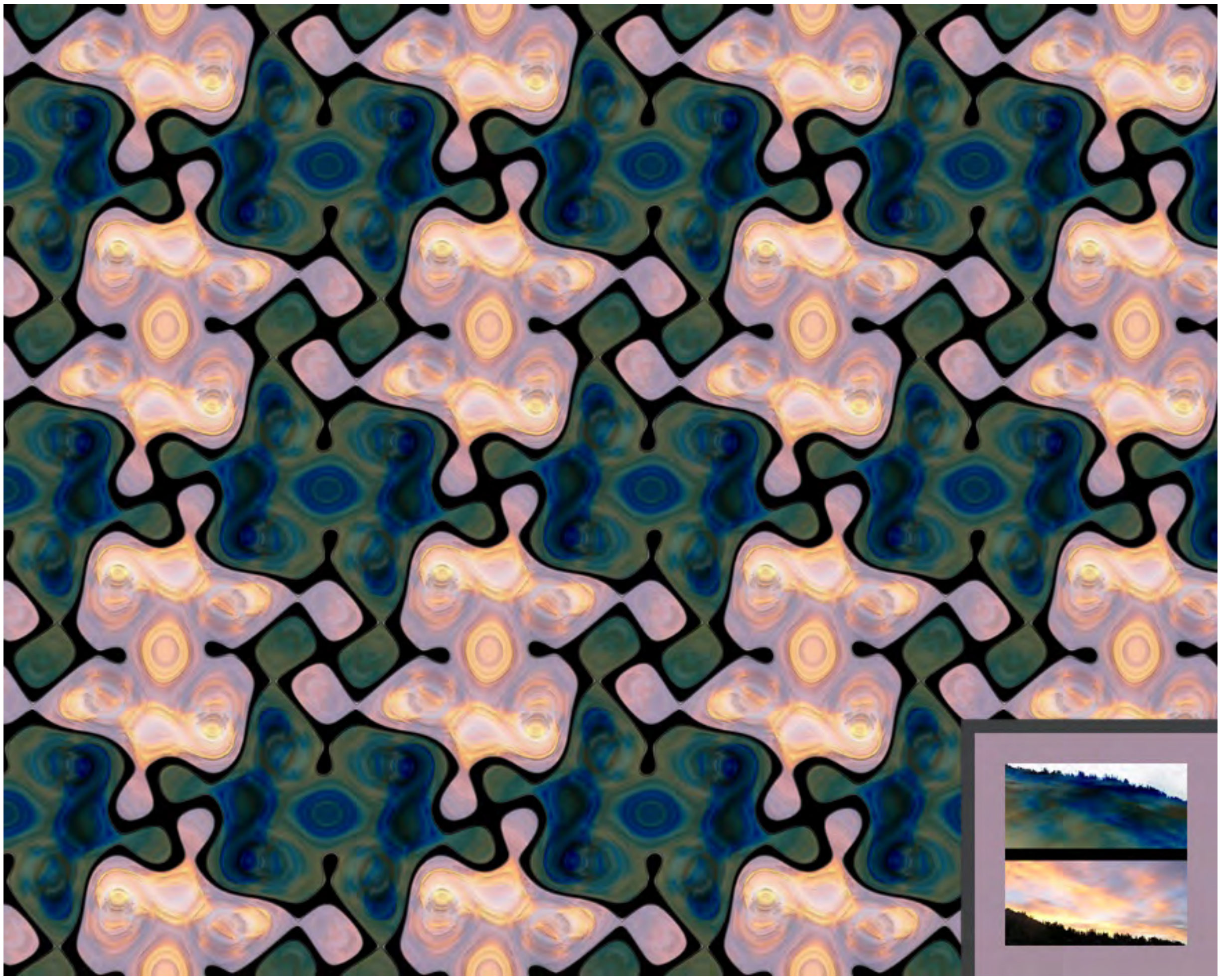


Puzzle Pieces from a Sierra Sunset

digital print on aluminum, 2012, 20” by 16”

Photographs with rather minimal variation (and artistic value) can turn into beautiful patterns. Can you find the color-reversing symmetry? If you turn the pink puzzle pieces 90°, they match the green ones with the same shapes but opposite colors.

The inset at the lower right shows that my source photograph is actually a collage, combining the original sunset photo with its negative, rotated upside down. Again, the black band is considered a neutral color to separate positive and negative colors.

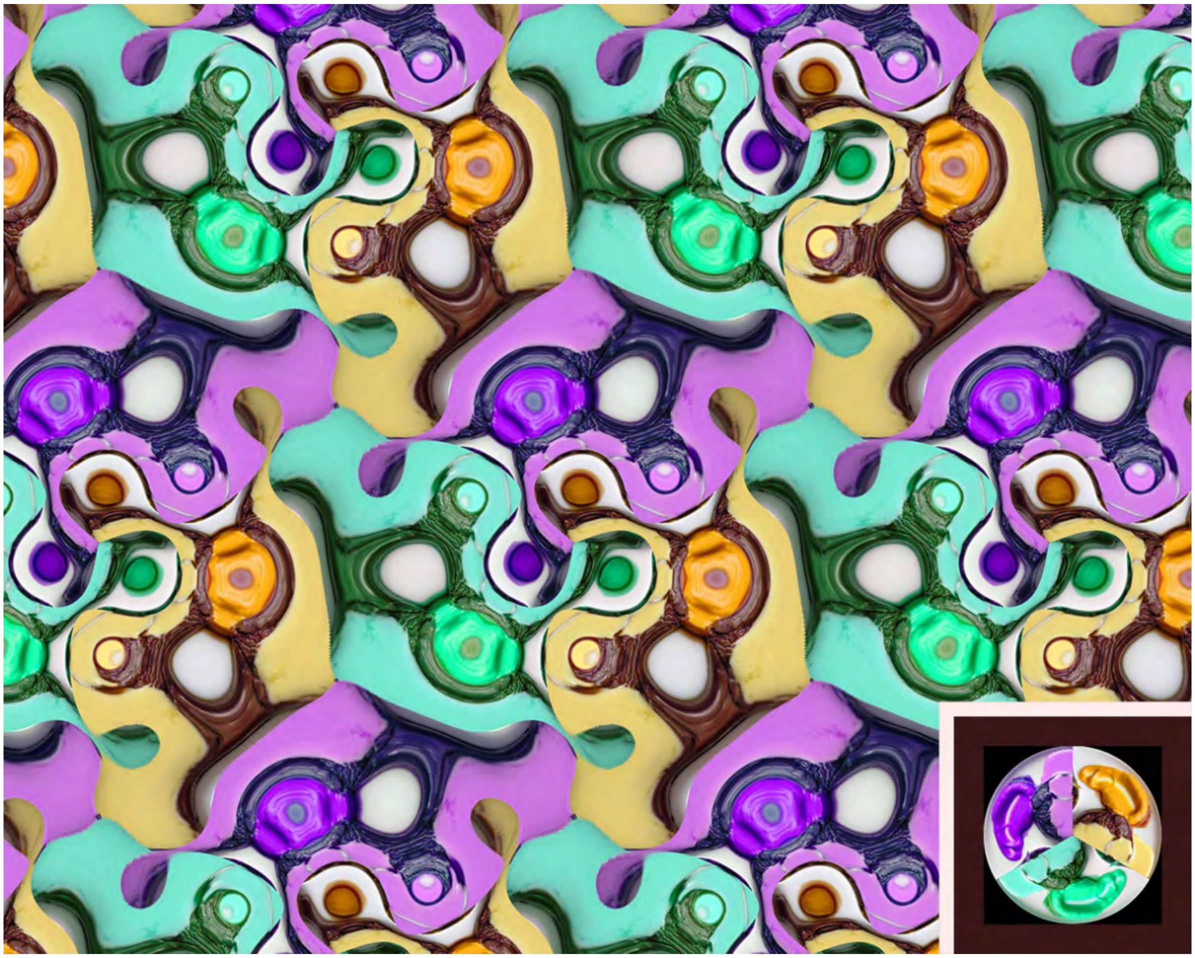


A Color-Turning Cartoon from a Steak in Perth

digital print on aluminum, 2012, 20” by 16”

Color-reversing patterns are also called *2-color patterns*, because you can make them with just two colors. Why stop at two?

First I turned a photo (of a beautiful steak and a smear of yams) into a collage where the colors shift by a third of the hue wheel when you turn it by 120°. Then I used a particular mathematical formula to make a pattern where the whole pattern has that same property: You can turn it 120° about various points and all the colors shift hue perfectly. Of course, it’s crazy to look at.



Symmetry Variations

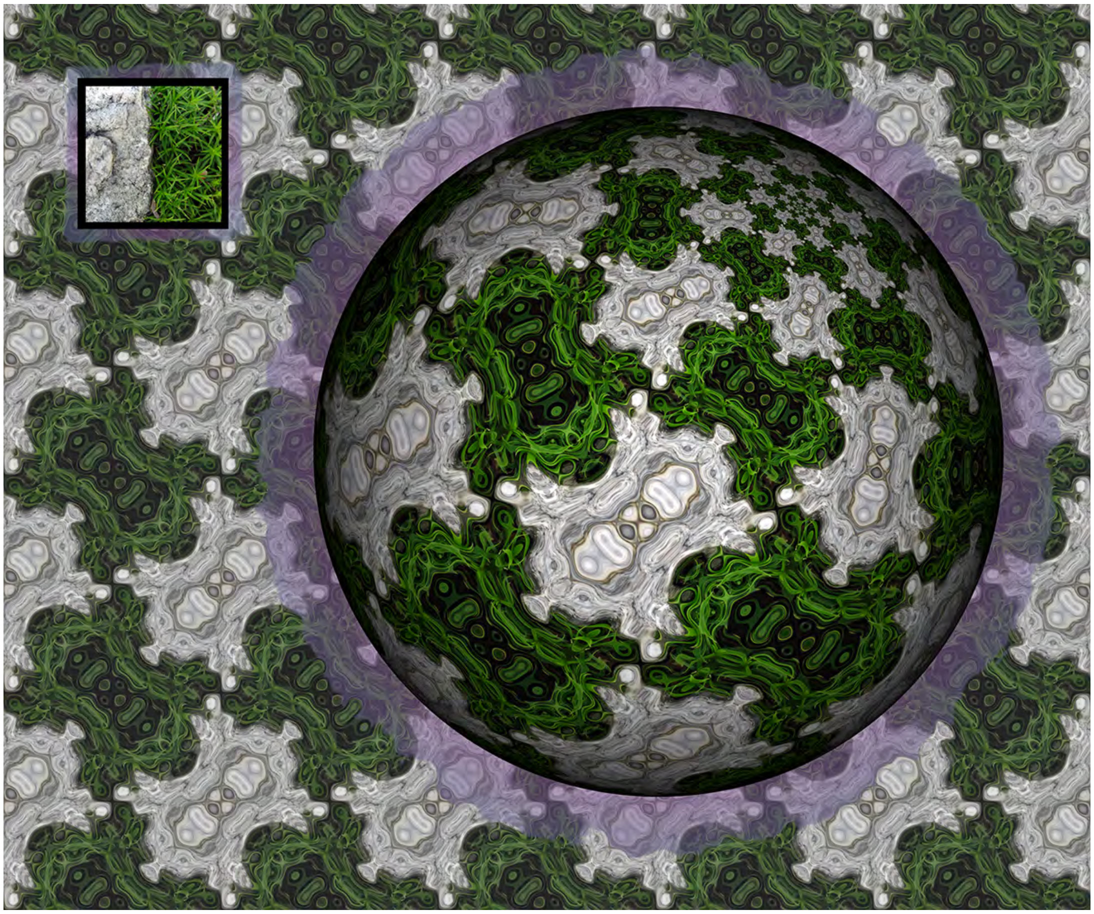
Plane symmetry was my first symmetry love, but it is only the beginning of the long story of patterns. There is great artistic potential in opening the door to patterns whose repetitions are not tied to rigid motions of the plane. For instance, allowing dilation symmetry leads to families of spiraling patterns. Expanding our concept of *plane* to include other domains can lead us to patterns on the sphere or into the puzzling world of non-Euclidean geometry.

Mossy Frogs and Granite Bugs Spiral on a Globe

digital print on aluminum, 2015, 24” by 20”

The pattern in the background of this image does not exactly have color-reversing symmetry. The source photograph of granite and moss is only vaguely color-reversing when you turn it upside down: the greens turn into grays. Still, when I used it with a formula that would yield color-reversing symmetry, it led to the two similar-but-different shapes: frogs and bugs. Then I wound the pattern onto a sphere in a spiral pattern.

I added the purple haze by hand, using Photoshop.

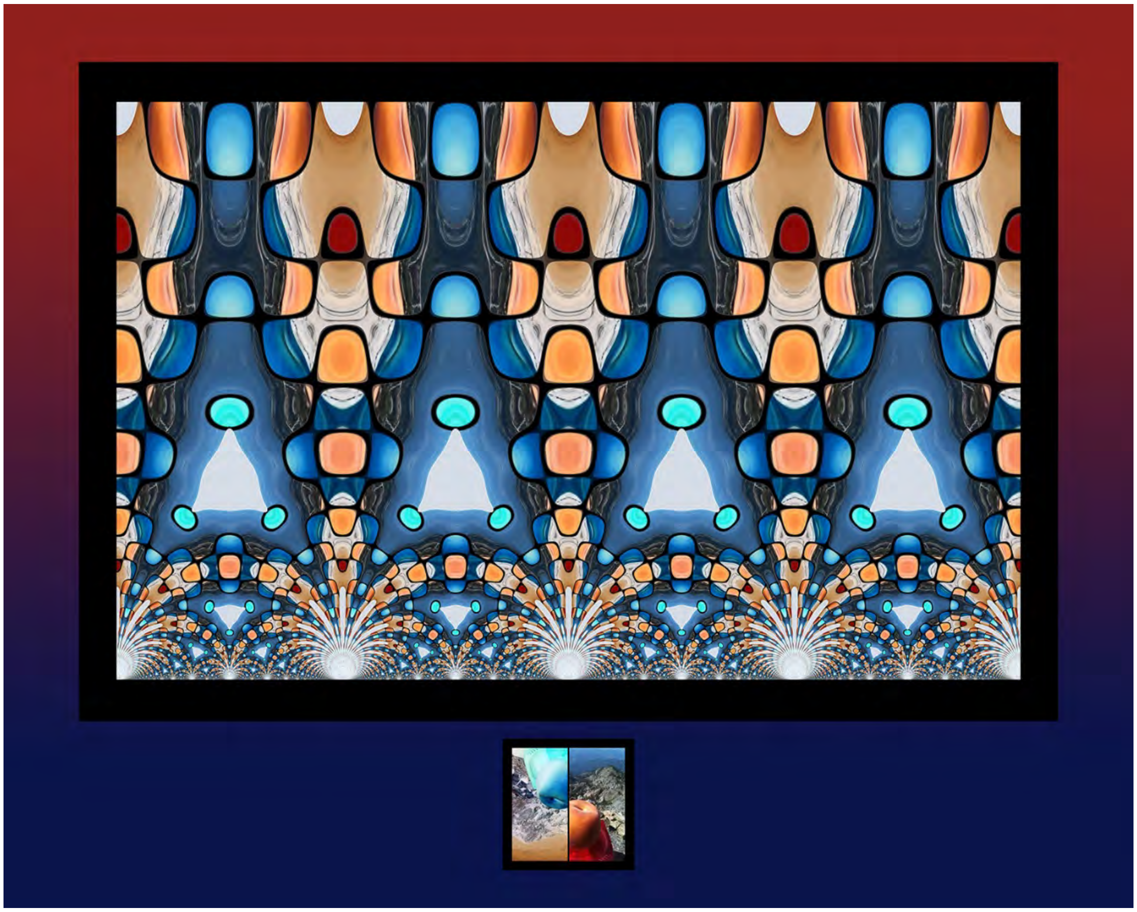


Hyperbolic Peach

digital print on aluminum, 2014, 30” by 24”

Think of non-Euclidean geometry as describing the properties of a toy universe, where there’s a distinct but inaccessible edge along the bottom. The reason you can’t reach that edge is that you and all your matter shrink in a predictable way as you drop toward it.

Find in the image the gray-blue triangles with bright dots at their corners; in this world of the shrinking ruler, they are all the same size!



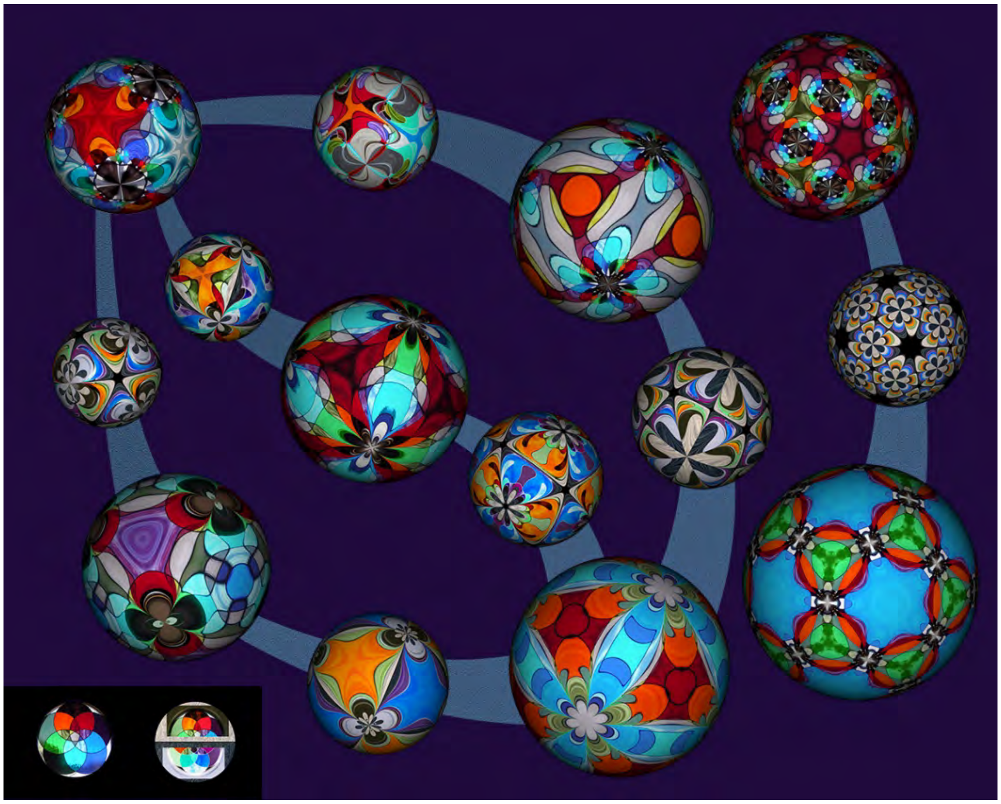
A Polyhedral Sampler

digital print on aluminum, 2015, 30” by 24”

The story of symmetry and color-reversing symmetry on the sphere is quite similar to the story on the Euclidean plane. It’s just a sort of round plane. The symmetries are those of the Platonic solids: tetrahedron, cube/octahedron, and dodecahedron/icosahedron.

There are actually seven different types of polyhedral symmetry, allowing different mirror symmetries, as exemplified by the seven larger globes in the image.

The smaller globes show color-reversing symmetry. With any color-reversing pattern we can think about what symmetries would be present if we took a color and its negative to be the same thing, as if anti-symmetries were actual symmetries. On each blue path, the smaller globe has the same symmetry type as the globe at the start of the path (moving down and to the right), and would have the symmetry type of the globe at the end if you were to identify colors with their negatives.



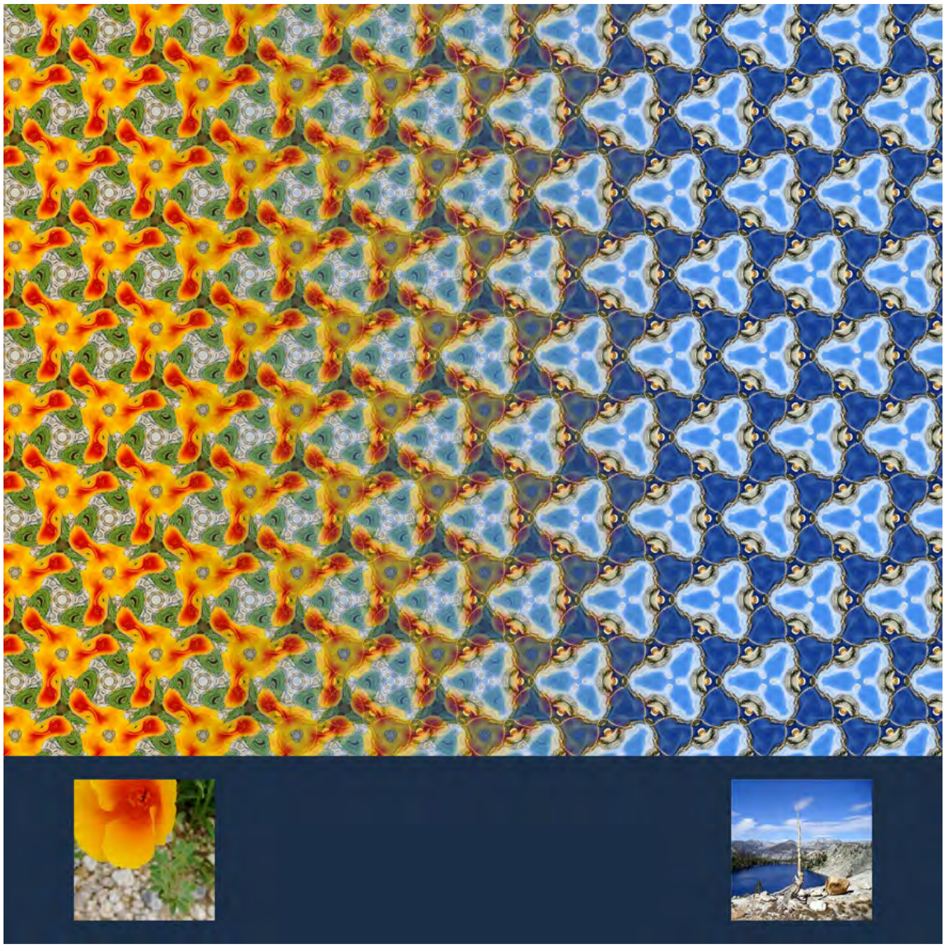
Metamorphosis: Homage to M.C. Escher

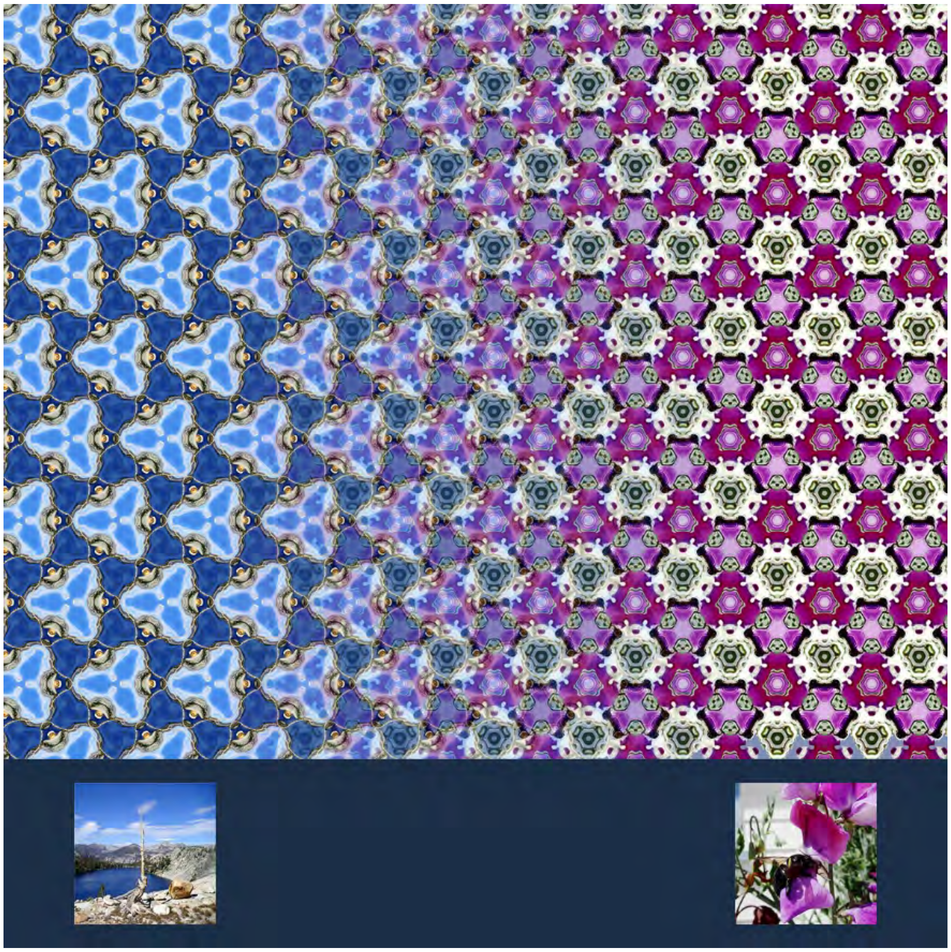
set of four digital prints on aluminum, 2016, each 20” by 20”

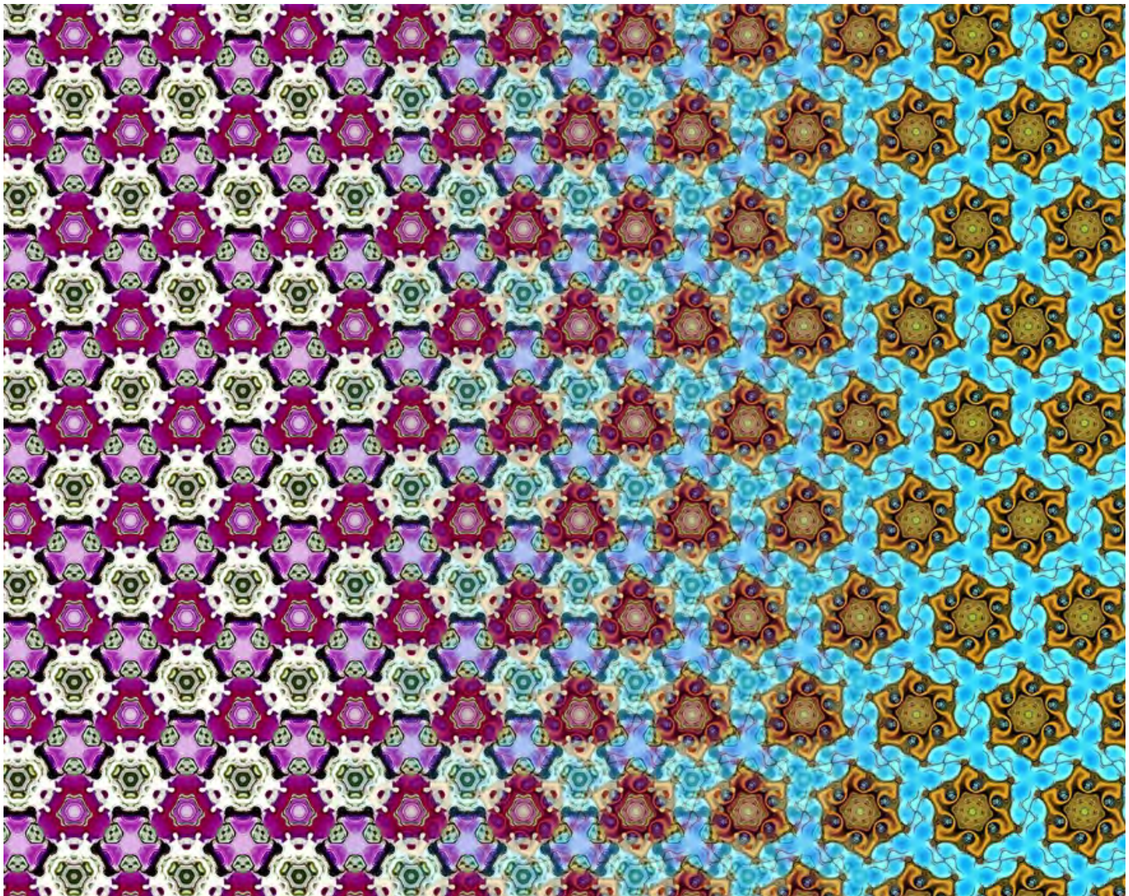
Everyone working in symmetry owes a debt to M.C. Escher, whose wonderful figurative drawings illustrate virtually every type of known plane symmetry. This set of four panels is meant to honor Escher’s great *Metamorphosis* series, woodcut prints with the largest almost 7 meters long.

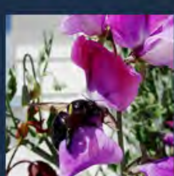
In *Metamorphosis II,* Escher’s magic turns lizards into bees into fish into birds; and all by hand. My technique for morphing requires much less cleverness: I simply blend from one pattern into another.

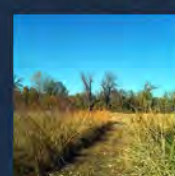
In this set of panels, I show every type of wallpaper symmetry with either 3-fold or 6-fold rotational symmetry, making this something of a wallpaper sampler. (Which pattern has the same type as “Autumn Moths”?)

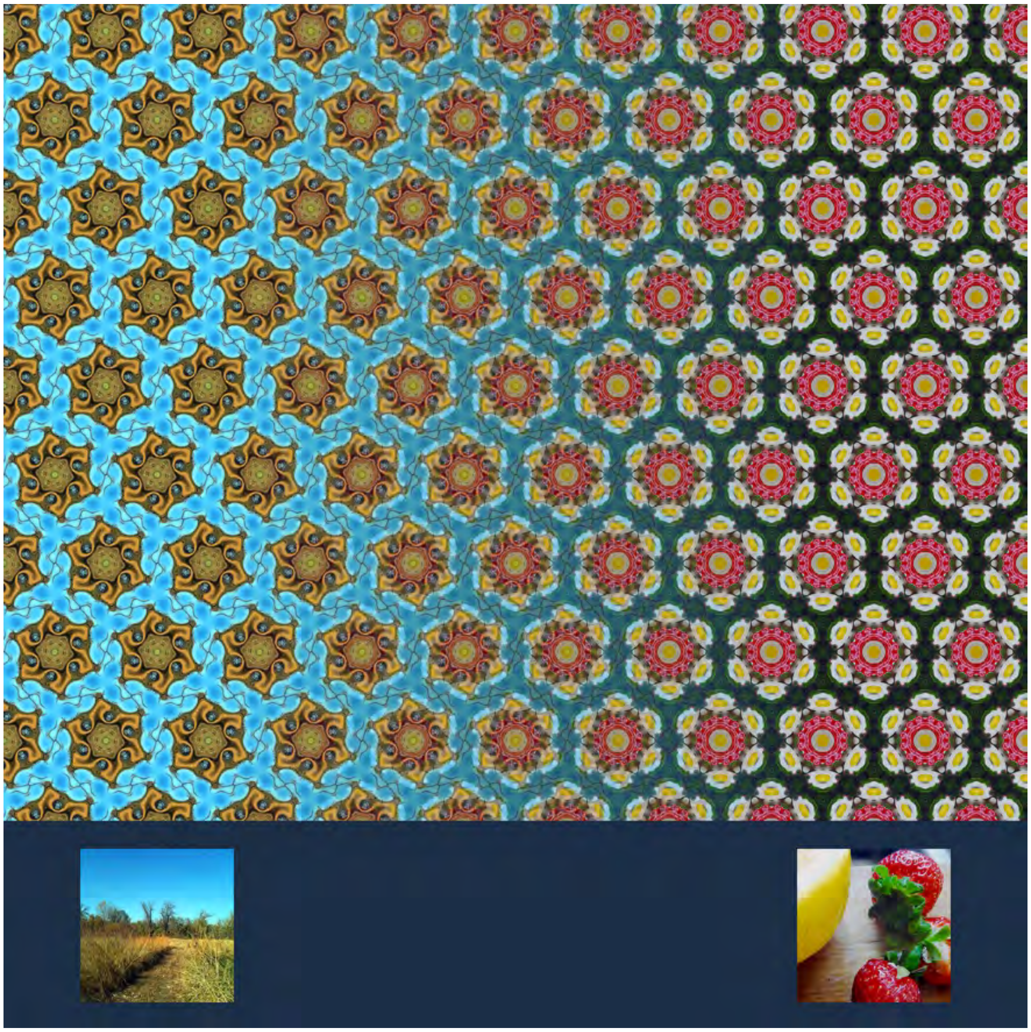












Imaginary Landscapes

For many years, I worked in the abstract, making patterns for their own sake. Recently I entered a figurative phase, creating images that blur boundaries between the real and mathematical worlds.

The technique of *ray-tracing* starts with a virtual scene populated by objects that are objects only in the sense of having a precise mathematical description. Elaborate software routines compute images by tracking the rays of virtual light that bounce off virtual objects in the scene to reach a virtual camera.

Ray-tracing is one of the principle techniques for creating digital effects in modern movies, where the intent is to deceive. My intention is far from photorealism. I invite you into my own personal realm where the real and mathematical dance together.

The Temple of the Peach

digital print on aluminum, 2016, 20” by 16”

Who visits the Temple of the Peach? My first step in going there was to paste the Hyperbolic Peach pattern that I used to illustrate non-Euclidean geometry around the inside of a cylinder, the temple wall.

The giant globe holder with its peach medallion holds a sphere decorated with another pattern made from the same peach photograph, which is propped on the floor. Photoshop allows me to tell that floor that it should be highly reflective. I had to let the ray-tracing software run for a day to produce this scene.

A Platonic Regatta

digital print on aluminum, 2016, 24” by 20”

The three windsails racing down Huntington Lake exemplify the (rotational) symmetries of the Platonic solids: icosahedron/dodecahedron, cube/octahedron, and tetrahedron. The icosahedron is painted with a picture of sweet peas, the cube with a bright marigold. The tetrahedron was too twisted to paint, so I used simple primary colors. Reflections on the water are a trick of ray-tracing.

They Arrive…

digital print on aluminum, 2016, 24” by 20”

Balls painted with polyhedral patterns waft over the mountain to land by moonlight on Upper Twin Lake in the Sierra Nevada. Where do they come from? How often do they appear?

The globes arrive in pairs, though not in order. Two have the symmetry of the rigid motions of the icosahedron; there are two each with octahedral/cube and tetrahedral symmetries.

A Cherry Donut

digital print on aluminum, 2016, 24” by 20”

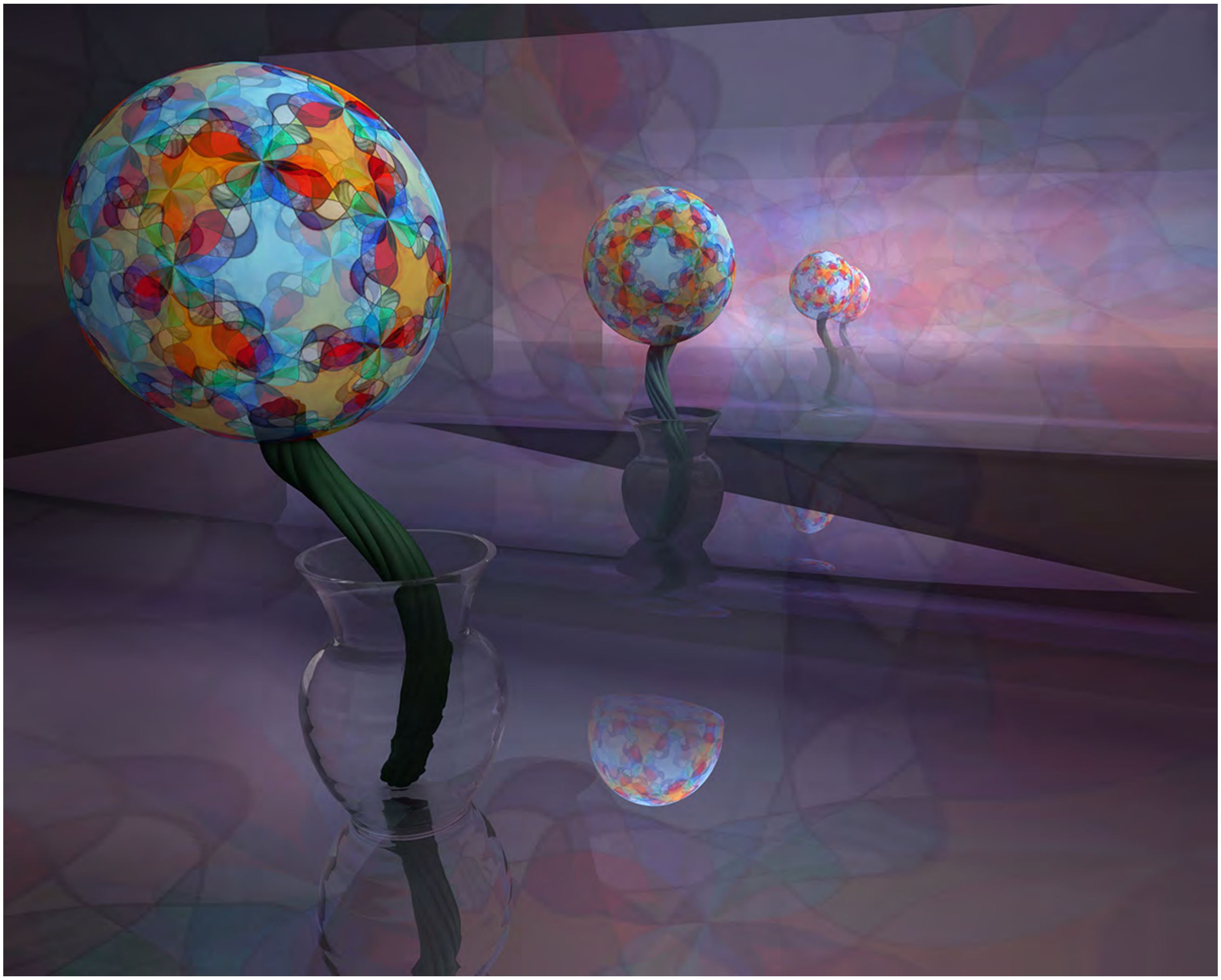
Any wallpaper pattern can be wound around a donut, though with some distortion since the donut bulges on the outside and squeezes on the inside.

The tilted wallpaper pattern, made from the photograph of cherries and chard on the wall, had to be scaled correctly to make it match at the donut seams. In a surreal effect, it almost seems that that pattern on the donut is climbing up onto the wall.

Icosahedral Lampflower

digital print on aluminum, 2016, 20” by 16”

This image tests the limits of ray-tracing, with a virtual lamp inside a partially transparent, icosahedrally painted globe placed atop a pumpkin stem in a crystal vase. Two opposing slightly curved reflective walls create a festival of reflections and projections. The source of the colors is my now-familiar stained glass window.

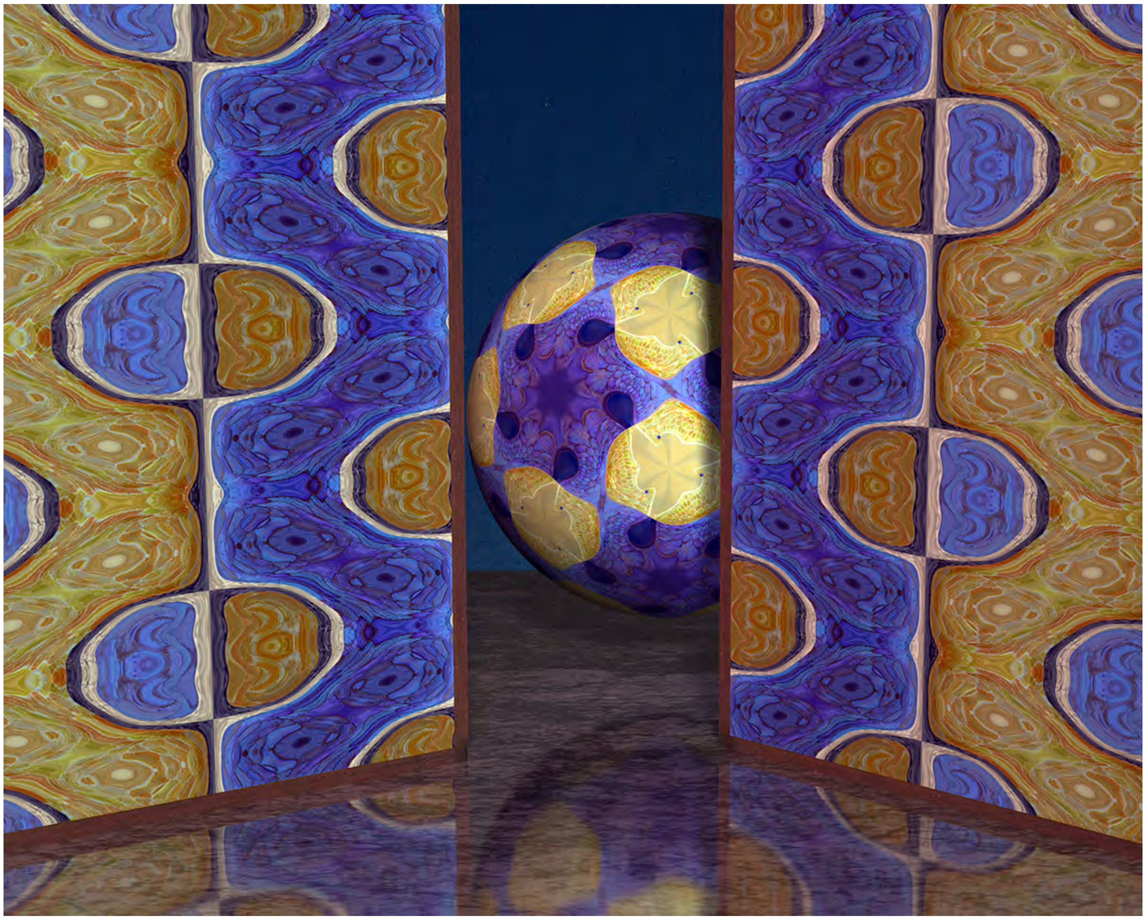


When the Butterfly Gate Opens

digital print on aluminum, 2016, 20” by 16”

This scene evokes Neal Stephenson’s novel *Anathem*, which describes isolated communities of mathematicians living apart from society. The gates of these monasteries open, depending on the community, every year, ten years, hundred years, or, for an enigmatic few, every thousand years.

Before the virtual lighting altered them, the colors of both patterns, gates and globe, were the same, drawn from a photograph (and negative) of a medallion of blue Morpho butterfly wings that I inherited from an ancestor. The pattern on the gates shows color-reversing wallpaper symmetry; the pattern on the floating globe is icosahedral.



Starry Night of Fire

digital print on aluminum, 2016, 20” by 16”

A spiral of fire stars shines a reflection on Huntington Lake at night. The globe floating on the lake glows with the same pattern, but in reverse: When we view the same pattern from the outside of the smaller globe, the 6-fold stars spin the opposite way from the ones on the inside of the dome of the heavens.