CS/Math 166 (10:30 AM)—Winter 2018 — Final Exam (**Two** hours)
D.C. Smolarski, S.J. — Monday, March 19, 2018 (9:10 AM)

NOTE 1: Three $8\frac{1}{2} \times 11$ sheets of paper are permitted with notes on it. Calculators may be used to assist in algebraic computations, but all major analytical steps must be provided for an answer to receive full credit.

NOTE 2: *RESERVE THE FIRST PAGE OF YOUR GREEN-BOOK* for the answers to the first 6 (multiple-choice) problems. Only the correct answer is needed. All these 6 problems are designed to have only ONE correct answer. Please put all these 6 answers on the same (first) page. Use other pages to record your computational work.

NOTE 3: Please keep the grader happy by starting every problem on a **new side of a page!**

1. (7) Which of the following functions is difficult to compute accurately for x near 1 but not equal to 1?
   (a) $\dfrac{1}{1+\ln x}$ (b) $(1-x^2)^{1/2}$ for $|x| < 1$
   (c) e^x (d) $\ln x + 1$

2. (7) Let $f(x)$ be a function for which the root r is to be computed, $f(r) = 0$. Assume f has a continuous derivative so that Newton's (the Newton-Raphson) method can be used.
   (a) Newton's method always converges quadratically to r if f is a quadratic polynomial and if x_0 (not equal to r) is sufficiently close to r.
   (b) Newton's method always converges faster than the method of bisection.
   (c) To get the same accuracy, Newton's method always requires less total work than the secant method if f is a polynomial of degree 2 or more.
   (d) Newton's method generally converges more rapidly than the secant method, although the total work for Newton's method may be greater.

3. (7) Let A be a nonsingular tridiagonal matrix of n rows by n columns. The efficient solution of $Ax = b$
   (a) takes $O(n)$ arithmetic operations. (b) takes $O(n^2)$ arithmetic operations.
   (c) takes $O(n^3)$ arithmetic operations. (d) requires b to be sparse.

4. (7) For the solution of $Ax = b$, A nonsingular, by Gaussian elimination,

   (a) pivoting reduces the number of arithmetic operations.

   (b) pivoting is always necessary.

   (c) pivoting is generally necessary if certain elements of b are large relative to other elements.

   (d) the work required to solve $Ax = c$ could be much less if $Ax = b$ has already been solved.

5. (7) Consider the differential equation, $x + 2xx' + x' = 0$. Then a form suitable for the solution by a Runge-Kutta method

   (a) depends on the order of the method. (b) is $x = \dfrac{-x'}{1+2x'}$.

   (c) is $x = \dfrac{x'}{1+2x'}$. (d) is $x' = \dfrac{-x}{1+2x}$.

6. (7) If it takes two seconds for a computer to solve a system of 100 linear equations through Gaussian elimination, about how long would it take the same computer to solve a system of 500 equations by the same code?

   (a) 125 sec. (b) 250 sec. (c) 25 sec. (d) 50 sec. (e) (other)= _____ sec.

   Show reasons for your answer.

7. (20) Transform the following linear system into a set of "normal equations."

$$\begin{array}{rcrcrcr} 3x & + & 4y & - & z & = & 0 \\ x & + & y & & & = & 2 \\ & & 2y & + & 3z & = & 4 \end{array}$$

8. (20) Using Gaussian quadrature with (a) $n = 2$ and then with (b) $n = 3$, evaluate

$$\int_{-1}^{1} x^5 + 2x^2 - x + 1dx.$$

GAUSSIAN QUADRATURE TABLE		
n	points	weights
2	$\pm\sqrt{1/3}$	1
3	$\pm\sqrt{3/5}$	5/9
	0	8/9

9. (20) Note that $\int \ln x dx = x \ln x - x$ (via integration by parts). Develop a new quadrature formula as follows:

$$\int_{1}^{2} (\ln x) f(x) dx = w f(\frac{3}{2}).$$

   We want this formula to be *exact* if $f(x)$ is a polynomial of degree 0. Find w.

10. (18) Change the third order ODE $y''' + 3y'' + t^2y' - yt^2 = 2$ where $y(0) = 1, y'(0) = 1, y''(0) = 3$ into a coupled system of first order ODEs. (Do *not* actually solve the ODE [system].)

11. (20) Using Simpson's 1/3 rule, how many panels are needed to obtain accuracy of 0.0001 if integrating the function $f(x) = x^5 + 4x^4$ from $x = 1$ to $x = 3$.

12. (24) "Solve" the ODE (BVP), $y'' + t^2y = 2t$, by discretization techniques given $y(1) = 2, y(3) = 2$ with $h = 1/2$. By "solve", you should set up the appropriate linear system in matrix form, but you do *not* have to actually solve the linear system.

13. (18) Use Richardson's extrapolation to obtain a better approximation, given the following information: using the trapezoidal rule with the number of panels being 5 and 10 we obtained approximations 1.02 and 1.005 respectively.

14. (18) Let $T(h)$ be the approximation to the integral $\int_0^1 f(x)dx$ using the trapezoidal rule with panels of equal width h. Let $f(x)$ be given by the table

x	0	1/4	1/2	3/4	1
y	0	3	4	3/2	2

Find $T(1)$, $T(1/2)$, and $T(1/4)$.

200 points total.