Numerical Analysis

Extra Topic III
2-Dimension Metal Plate (with hole)

Heat Equation

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Solving a PDE as a Linear System

Look at the Poisson equation $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = Q(x, y)$. If one includes a partial derivative with respect to t, one has the heat equation that includes a time component. However, the time component can be zero, in which case, the heat equation reduces to the Poisson equation.

Example: Rectangular Metal Plate

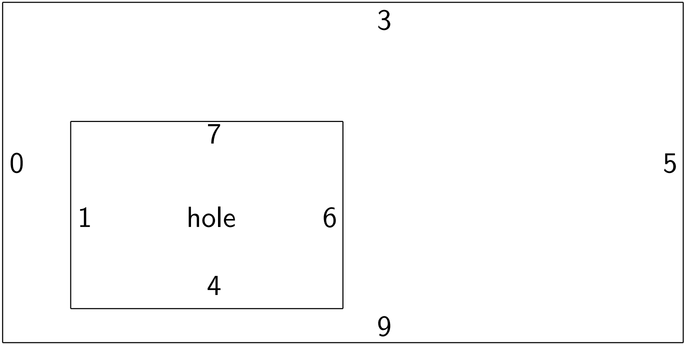
Assume we have a rectangular metal plate, 101 units wide and 51 units high.

Suppose we have a hole in the plate, from rows 17 to 46 and from columns 12 to 51 (i.e., a rectangle about 30 tall by 40 wide).

– Suppose the "temperature" on the left edge of the plate is constrained to be 0, on the top edge is 3, on the bottom edge is 9 and on the right edge is 5.
–Suppose the "temperature" on the left edge of the hole is constrained to be 1, on the top edge is 7, on the bottom edge is 4 and on the right edge is 6.

We would like to know the variation of temperature on the metal plate itself.

Metal Plate Depiction



We can discretize the “plate” into discrete points and transform the PDE into a linear system to be solved, iteratively.

Afterwards, we can “color code” the values and render the heat distribution as a color 2-D plot. (This can be easily done via a Matlab command.)

COLORED OUTPUT PLOT

Note: that because of the automatic scaling in Matlab, the horizontal and vertical scales are not the same.