

1. Euler's method gives the approximations in the following tables.

5.2

(a)

i	t_i	w_i	$y(t_i)$
1	0.500	0.0000000	0.2836165
2	1.000	1.1204223	3.2190993

(b)

i	t_i	w_i	$y(t_i)$
1	2.500	2.0000000	1.8333333
2	3.000	2.6250000	2.5000000

(c)

i	t_i	w_i	$y(t_i)$
1	1.250	2.7500000	2.7789294
2	1.500	3.5500000	3.6081977
3	1.750	4.3916667	4.4793276
4	2.000	5.2690476	5.3862944

(d)

i	t_i	w_i	$y(t_i)$
1	0.250	1.2500000	1.3291498
2	0.500	1.6398053	1.7304898
3	0.750	2.0242547	2.0414720
4	1.000	2.2364573	2.1179795

5. Euler's method gives the approximations in the following tables.

(a)

i	t_i	w_i	$y(t_i)$
2	1.200	1.0082645	1.0149523
4	1.400	1.0385147	1.0475339
6	1.600	1.0784611	1.0884327
8	1.800	1.1232621	1.1336536
10	2.000	1.1706516	1.1812322

(b)

i	t_i	w_i	$y(t_i)$
2	1.400	0.4388889	0.4896817
4	1.800	1.0520380	1.1994386
6	2.200	1.8842608	2.2135018
8	2.600	3.0028372	3.6784753
10	3.000	4.5142774	5.8741000

(c)

i	t_i	w_i	$y(t_i)$
2	0.400	-1.6080000	-1.6200510
4	0.800	-1.3017370	-1.3359632
6	1.200	-1.1274909	-1.1663454
8	1.600	-1.0491191	-1.0783314
10	2.000	-1.0181518	-1.0359724

(d)

i	t_i	w_i	$y(t_i)$
2	0.2	0.1083333	0.1626265
4	0.4	0.1620833	0.2051118
6	0.6	0.3455208	0.3765957
8	0.8	0.6213802	0.6461052
10	1.0	0.9803451	1.0022460

9. Euler's method gives the approximations in the following table.

(a)

i	t_i	w_i	$y(t_i)$
1	1.1	0.271828	0.345920
5	1.5	3.18744	3.96767
6	1.6	4.62080	5.70296
9	1.9	11.7480	14.3231
10	2.0	15.3982	18.6831

(b) Linear interpolation gives the approximations in the following table.

t	Approximation	$y(t)$	Error
1.04	0.108731	0.119986	0.01126
1.55	3.90412	4.78864	0.8845
1.97	14.3031	17.2793	2.976

(c) $h < 0.00064$

1. Taylor's method of order two gives the approximations in the following tables.

(a)

t_i	w_i	$y(t_i)$
0.50	0.12500000	0.28361652
1.00	2.02323897	3.21909932

(b)

t_i	w_i	$y(t_i)$
2.50	1.75000000	1.83333333
3.00	2.42578125	2.50000000

(c)

t_i	w_i	$y(t_i)$
1.25	2.78125000	2.77892944
1.50	3.61250000	3.60819766
1.75	4.48541667	4.47932763
2.00	5.39404762	5.38629436

(d)

t_i	w_i	$y(t_i)$
0.25	1.34375000	1.32914981
0.50	1.77218707	1.73048976
0.75	2.11067606	2.04147203
1.00	2.20164395	2.11797955

- 5.3
5. (a) The number of infectives is $y(30) \approx 80295.7$.
- (b) The limiting value for the number of infectives for this model is $\lim_{t \rightarrow \infty} y(t) = 100,000$.

5.6 Multistep Methods

g a)

t_i	w_i	$y(t_i)$
0.2	0.0269059	0.0268128
0.4	0.1510468	0.1507778
0.6	0.4966479	0.4960196
0.8	1.3408657	1.3308570
1.0	3.2450881	3.2190993

b)

t_i	w_i	$y(t_i)$
2.2	1.3666610	1.3666667
2.4	1.6857079	1.6857143
2.6	1.9749941	1.9750000
2.8	2.2446995	2.2444444
3.0	2.5003083	2.5000000

11.6)

1. The Linear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
1	0.5	0.82432432	0.82402714

(b)

i	x_i	w_{1i}	$y(x_i)$
1	0.25	0.3937095	0.3936767
2	0.50	0.8240948	0.8240271
3	0.75	1.337160	1.337086

2. The Linear Shooting Algorithm gives the results in the following tables.

(a)

i	x_i	w_{1i}	$y(x_i)$
1	0.78539816	-0.28245222	-0.28284271

(b)

i	x_i	w_{1i}	$y(x_i)$
1	$\pi/8$	-0.31541496	-0.31543220
2	$\pi/4$	-0.2828507	-0.282842712
3	$3\pi/8$	-0.20718437	-0.20719298

11.3

1. The Linear Finite-Difference Algorithm gives following results.

(a)

i	x_i	w_{1i}	$y(x_i)$
1	0.5	0.83333333	0.82402714

(b)

i	x_i	w_{1i}	$y(x_i)$
1	0.25	0.39512472	0.39367669
2	0.5	0.82653061	0.82402714
3	0.75	1.33956916	1.33708613

(c) Extrapolation gives

$$y\left(\frac{1}{2}\right) \approx \frac{4(0.82653061) - 0.83333333}{3} = 0.82426304$$

3. The Linear Finite-Difference Algorithm gives the results in the following tables.

(a)

i	x_i	w_i	$y(x_i)$
2	0.2	1.018096	1.0221404
5	0.5	0.5942743	0.59713617
7	0.7	0.6514520	0.65290384

(b)

i	x_i	w_i	$y(x_i)$
5	1.25	0.16797186	0.16762427
10	1.50	0.45842388	0.45819349
15	1.75	0.60787334	0.60777401

(c)

i	x_i	w_{1i}	$y(x_i)$
3	0.3	-0.5183084	-0.5185728
6	0.6	-0.2192657	-0.2195247
9	0.9	-0.0405748	-0.04065697

(d)

i	x_i	w_{1i}	$y(x_i)$
3	1.3	0.0654387	0.0655342
6	1.6	0.0773936	0.0774595
9	1.9	0.0305465	0.0305621

(20)

1. The Poisson Equation Finite-Difference Algorithm gives the following results.

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
1	1	0.5	0.5	0.0	0
1	2	0.5	1.0	0.25	0.25
1	3	0.5	1.5	1.0	1

2. The Poisson Equation Finite-Difference Algorithm gives the following results.

i	j	x_i	y_j	w_{ij}	$u(x_i, y_i)$
1	1	1.33333333	0.33333333	0.6348043	0.6359888
1	2	1.33333333	0.66666667	0.7985001	0.7985077
2	1	1.66666667	0.33333333	1.0599924	1.0687200
2	2	1.66666667	0.66666667	1.1698208	1.1700713

3. The Poisson Equation Finite-Difference Algorithm gives the following results.

(a) 30 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
2	2	0.4	0.4	0.1599988	0.16
2	4	0.4	0.8	0.3199988	0.32
4	2	0.8	0.4	0.3199995	0.32
4	4	0.8	0.8	0.6399996	0.64

(b) 29 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
2	1	1.256637	0.3141593	0.2951855	0.2938926
2	3	1.256637	0.9424778	0.1830822	0.1816356
4	1	2.513274	0.3141593	-0.7721948	-0.7694209
4	3	2.513274	0.9424778	-0.4785169	-0.4755283

(c) 126 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
4	3	0.8	0.3	1.2714468	1.2712492
4	7	0.8	0.7	1.7509414	1.7506725
8	3	1.6	0.3	1.6167917	1.6160744
8	7	1.6	0.7	3.0659184	3.0648542

(d) 127 iterations required:

i	j	x_i	y_j	$w_{i,j}$	$u(x_i, y_j)$
2	2	1.2	1.2	0.5251533	0.5250861
4	4	1.4	1.4	1.3190830	1.3189712
6	6	1.6	1.6	2.4065150	2.4064186
8	8	1.8	1.8	3.8088995	3.8088576