VECTORS 12.1 – 12.2

- Math'l concept with both magnitude and direction
(Scalar only has magnitude)

a unit vector has length 1.

$\vec{i}$ unit vector in positive x-direction
$\vec{j}$ unit vector in positive y-direction

For 3D need:

$\vec{k}$ unit vector in positive z-direction.

Right hand rule: Right hand on x-axis with fingers curled toward y-axis, then thumb points in z-axis (positive).

Length of vector is computed used Pythagorean Thm.

i.e. Given $\vec{v} = a\vec{i} + b\vec{j}$, Then $|\vec{v}| = \sqrt{a^2+b^2}$

Direction of a vector is a unit vector obtained:

$$dir(\vec{v}) = \frac{\vec{v}}{|\vec{v}|}$$

DOT PRODUCT 12.3

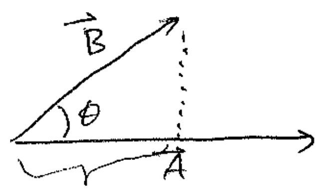
→ Gives scalar value.

$$\vec{A} \circ \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$ where θ is angle between $\vec{A}$ and $\vec{B}$

$$= a_1 b_1 + a_2 b_2 (+ a_3 b_3)$$

→ $\vec{A} \circ \vec{B} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta$ is $90° \Rightarrow \vec{A} \perp \vec{B}$

→ PROJECTION



length $\text{proj}_{\vec{A}} \vec{B}$

NOTE $\dfrac{\text{length proj } \vec{B}}{|\vec{B}|} = \cos\theta$

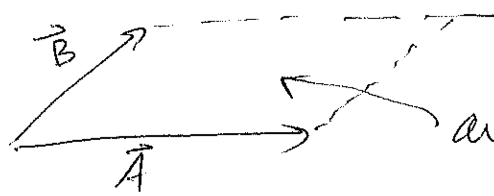
$$\Rightarrow \text{length proj } \vec{B} = |\vec{B}| \cos\theta = \cancel{|\vec{B}|} \frac{\vec{A} \circ \vec{B}}{|\vec{A}|\,\cancel{|\vec{B}|}} = \frac{\vec{A} \circ \vec{B}}{|\vec{A}|}$$

CROSS PRODUCT 12.4

→ Give vector value

$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \cdot \sin\theta \cdot \vec{n}$ where $\vec{n}$ is unit vector normal to plane formed by $\vec{A}, \vec{B}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



area of parallelogram is $|\vec{A} \times \vec{B}|$

EQS OF LINES AND PLANES

Given direction vector $A\vec{i} + B\vec{j} + C\vec{k}$
and point $P_0(x_0, y_0, z_0)$

a line parallel to vector through P_0

$$\frac{(x-x_0)}{A} = \frac{(y-y_0)}{B} = \frac{(z-z_0)}{C} = t$$

or $x = x_0 + At$, $y = y_0 + Bt$, $z = z_0 + Ct$

a plane with vector a NORMAL (perpendicular) containing P_0

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

E.G. Given $\vec{d} = 3\vec{i} + 2\vec{j} + 4\vec{k}$ and $P_0(1,2,1)$
line $\parallel \vec{d}$ through P_0 is

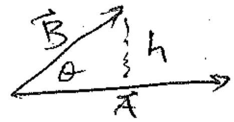
$$\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-1}{4}$$

plane $\perp \vec{d}$ containing P_0 is

$$3(x-1) + 2(y-2) + 4(z-1) = 0$$

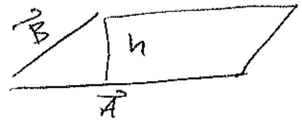
OTHER RELATED PROB'S

① $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta\,\overbrace{|\vec{n}|}^{1} = |\vec{A}||\vec{B}|\sin\theta$



$\frac{h}{|\vec{B}|} = \sin\theta \Rightarrow h = |\vec{B}|\sin\theta$

$\Rightarrow$



$\leftarrow$ Area $= |\vec{A}|\,h = |\vec{A}||\vec{B}|\sin\theta$
$= |\vec{A}\times\vec{B}|$.

② angle between 2 planes = angle between 2 normal vectors of plane

$\Rightarrow$ use dot prod to find angle

CYLINDERS + QUADRIC SURFACES 12-6

"CYLINDER" – any 2D curve moved in space || to 3rd axis is a "cylinder"

E.G. $x^2+y^2=1$ is a circle in 2D, but same eq represents a "right circular cylinder" in 3D (we imagine the circle moving up the z-axis)

→ $\frac{x^2}{1}+\frac{z^2}{4}=1$ ⇒ elliptical cylinder || y-axis

→ $z=\frac{y^2}{4}$ ⇒ parabolic cylinder || x-axis

QUADRIC SURFACES – a surface described by a quadratic in x, y, z.

→ To visualize, it is easiest to set one variable to a constant (slicing the surface by a plane) and looking at resulting eq.

$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z}{c}$$

if $x=a$ ⇒ $\frac{y^2}{b^2}-1=\frac{z}{c}$ ⇒ parabola

if $y=b$ ⇒ $1-\frac{x^2}{a^2}=\frac{z}{c}$ ⇒ parabola

if $z=c$ ⇒ $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ ⇒ hyperbola