HYPERBOLIC FUNCTIONS 7.3

$$\sinh u = \frac{e^u - e^{-u}}{2} \qquad \cosh u = \frac{e^u + e^{-u}}{2}$$

Other functions defined as with circular trig functions, e.g., $\tanh u = \frac{\sinh u}{\cosh u}$

Basic identity $\cosh^2 u - \sinh^2 u = 1$

Derivatives In general, results are same as for circular trig functions, EXCEPT for signs:

$$\left.\begin{aligned} \frac{d}{dx}(\sinh x) &= \cosh x \\ \frac{d}{dx}(\cosh x) &= \sinh x \end{aligned}\right\} \text{ all positive deriv's}$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\left.\begin{aligned} \frac{d}{dx}(\operatorname{sech} x) &= -\operatorname{sech} x \tanh x \\ \frac{d}{dx}(\operatorname{csch} x) &= -\operatorname{csch} x \coth x \end{aligned}\right\} \text{ all negative deriv's (reciprocal function)}$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

Integration

Follow from derivatives, e.g $\int \cosh u \, du = \sinh u + c$

Inverse hyperbolic functions

$$y = \arg\sinh x \iff \sinh y = x$$

$$\Rightarrow \frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

SEQUENCES 10.1

$a_1, a_2, \ldots a_n \ldots$ ← JUST ORDERED LIST OF NUMBERS

We can ask whether the sequence converges or diverges (alternates)

E.g. $1, 2, 3, 4, 5 \ldots$ div

$1, 1, 1, 1, 1 \ldots 1 \ldots$ converges $\to 1$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n} \ldots$ converges $\to 0$

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots \frac{1}{n^2} \ldots$ converges $\to 0$

$1, -1, 1, -1, \ldots (-1)^{n+1} \ldots$ div (alternates)

SERIES 10.2

Sum (infinite) of sequence of Numbers

$$S = \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots = 1$$

$$S = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots \to \infty$$

Geometric series

$$S = a + ar + ar^2 + \ldots + ar^n + \ldots$$

If $|r| < 1$, then

$$S = \frac{a}{1-r}$$

POWER SERIES 10.8

Used to convert arbitrary function into an infinite polynomial

2 type:

Taylor

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

Maclaorin

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

EXAMPLES

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

BINOMIAL SERIES

for $-1 < x < 1$

Maclaurin Series for $(1+x)^m$ is $1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$

$$= 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots$$

PARAMETRIC EQ'S 11.1

We use $\underline{2}$ eq's to describe $\underline{1}$ curve, using a parameter, usually $\underline{t}$ for $\underline{\text{time}}$.

$\therefore \quad x = x(t) \quad , \quad y = y(t)$

This is helpful to find $\underline{\text{when}}$ you are at a pt on a curve.

Sometimes we can combine 2 eqs to one and eliminate t,

e.g. $\quad x = 2t+1 \quad \Rightarrow \quad 2t = x-1 \Rightarrow t = \frac{x-1}{2}$

$\quad\quad y = t^2 \quad\quad \Rightarrow \quad y = \frac{(x-1)^2}{4}$

Deriv. $\quad$ Given $\quad x = f(t) \;, \; y = g(t) \qquad$ 11.2

$$\underline{\frac{dy}{dx}} = \frac{dy/dt}{dx/dt}.$$

Length along a curve

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

EG. $\quad x = \sin 2t \qquad y = \cos 2t \qquad 0 \le t \le 2\pi$

$\quad \frac{dx}{dt} = 2\cos 2t \qquad \frac{dy}{dt} = -2\sin 2t$

$$L = \int_0^{2\pi} \sqrt{4\cos^2 2t + 4\sin^2 2t}\, dt = \int \sqrt{4}\sqrt{\underbrace{\cos^2 2t + \sin^2 2t}_{1}}\, dt$$

$$= 2\int dt = 2t\Big]_0^{2\pi} = 4\pi - 0 = \underline{4\pi}$$

CONIC SECTIONS   11.6, A.3

$x^2+y^2=r^2$ ← circle centered at (0,0) with radius r

→ $(x-h)^2+(y-k)^2=r^2$ circle centered at (h,k)

$y=\frac{1}{4p}x^2$ ← parabola with vertex at (0,0) and focus at (0,p)

→ $(y-k)=\frac{1}{4p}(x-h)^2$ parabola with vertex at (h,k)

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ ← ellipse centered at (h,k)

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2}=1$ ← hyperbola centered at (h,k)

Eg. $4x^2-9y^2-16x-18y-29=0$

$4x^2-16x+\underline{16}\ -9y^2-18y\ \underline{\underline{-9}}=29+\underline{16}\ \underline{\underline{-9}}$

$4(x^2-4x+4)-9(y^2+2y+1)=36=6^2$

$4(x-2)^2-9(y+1)^2=36$

$\frac{(x-2)^2}{9}-\frac{(y+1)^2}{4}=1$

⇒ hyperbola with center (2,−1)

$x^2+4y^2-2x+16y+23=0$

$x^2-2x+1+4y^2+16y+16=-23+1+16=-6$

$(x-1)^2+4(y+2)^2=-6$ ← impossible

∴ although it looks like a conic, it is not.