CYLINDERS (12.6) p 740

After a plane, the next simplest 3-dimensional surface is what is called a cylinder. There are many different types of cylinders, not only the "right circular cylinder" (= can), which we are most familiar with.

In general, a cylinder is a surface generated by a straight line moving parallel to a given line (most often one of the axes) and passing through a given curve (which gives the cross-sectional shape).

Any 2-dimensional curve, when situated in a 3-dimensional context, represents a cylinder. In general, an equation with one of the 3 variables missing represents a cylinder.

EG 1 | EX 1 p 740 $y = x^2$

(in 3-dim) is a parabolic cylinder. If you fix a value of z, you get a parabola.

EG 2 cf p 744 # 3 $y^2 + 4z^2 = 4$

(in 3-dim) is an elliptic cylinder.

E6.

$$x^2 - z^2 = 1$$

is a hyperbolic cylinder.

cf p 744 #13

$$x^2 + y^2 = a^2$$

is a right circular cylinder.

QUADRIC SURFACES p 741

The next degree of difficulty of 3-dimensional curves and surfaces are those called QUADRIC SURFACES. The equations of these surfaces are quadratic in the variables.

One of the best ways to analyze (and identify) these surfaces is to take cross sections, i.e. hold one variable constant, look at the resulting equation and see what it represents.

<u>EG 1</u> $$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=a^2$$

If we hold <u>any</u> variable constant, we can easily see that the resulting equation is a circle. The 3-dimensional figure is a <u>sphere</u>. Center is (x_0, y_0, z_0) and radius $= a$.

<u>EG 2</u> (EX 2, p 741) also Table 12.1 p 743 #1

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

If we hold <u>any</u> variable constant, we get an ellipse. This is an <u>ellipsoid</u>. (egg-shaped)

(cf Old Senate Chambers, U.S. Capitol Bldg, Wash. DC)

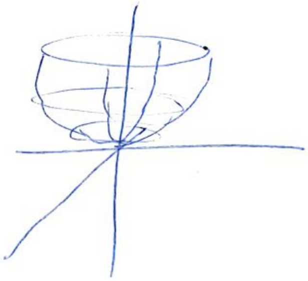
EG 3 (Table 12.1, p 743, #2)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

If we hold z constant, we get an ellipse. I.e. by cutting the figure perpendicular to (into) the z-axis, we get an ellipse as a cross section.

On the other hand, if we hold x or y constant, we get a parabola.

The surface is called an elliptic paraboloid (like an egg cup).



EG 4 (Table 12.1, p 743, #4)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

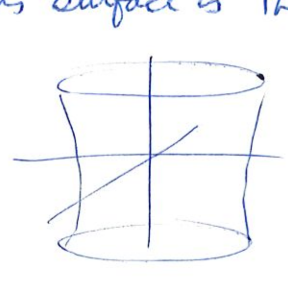
(NOTE sign diff with EG 2)

at $x=0$ (cutting the figure with the y-z plane) gives us $\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ which is a hyperbola.

at $y=0$ (xz plane), we get $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$, also a hyperbola

at $z=0$ (xy plane), we get $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, an ellipse.

This surface is the hyperboloid of one sheet



Tab 12.1 #5

EG 5 (Table 12.1 #5)

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

If $|z| < c$, this equation reduces to an impossibility since $-\frac{x^2}{a^2} - \frac{y^2}{b^2} < 0$ and $1 - \frac{z^2}{c^2} > 0$.

Therefore there must be a gap in the surface between $z = -c$ and $z = c$.

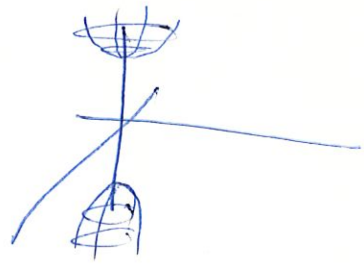
If $|z_0| \geq c$, we get $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z_0^2}{c^2} - 1$ which is an ellipse.

At $x = 0$, we have $\frac{z^2}{c^2} - \frac{y^2}{b^2} = 1$, a hyperbola.

At $y = 0$, we have $\frac{z^2}{c^2} - \frac{x^2}{a^2} = 1$, also an hyperbola.

This surface is called the hyperboloid of 2 sheets.

(Fig 15.10)



EG 6 (Ex 3, p 71/2) Table 12.1 #6

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}$$

If $x=0$, we get $\frac{y^2}{b^2} = \frac{z}{c}$, a parabola (concave up)

If $y=0$, we get $-\frac{x^2}{b^2} = \frac{z}{c}$, a parabola (concave down)

If z is constant, we get $\frac{y^2}{b^2} - \frac{x^2}{a^2} = d$, a hyperbola.

This is called a hyperbolic paraboloid or (commonly) a saddle.

The point at the origin is the "minimax" or saddle point, since going along the y-axis ($x=0$) you get a minimum while going along the x-axis ($y=0$) you get a maximum.