Math 13-3 (10:30 AM)—Fall, 2024 Midterm III (45 minutes)
D.C. Smolarski, S.J. Friday, November 8, 2024

NOTES:

#perfect scores per problem

1. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
2. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
3. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
4. **NO CALCULATORS!**

15/32

1. (18) Given $\vec{A} = 3\vec{\imath} - 2\vec{\jmath} + 4\vec{k}$ and $\vec{B} = 2\vec{\imath} + 5\vec{\jmath} - 4\vec{k}$,
   (a) find the angle between $\vec{A}$ and $\vec{B}$,
   (b) find the length of the projection of $\vec{A}$ onto $\vec{B}$.

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2. (18) Given $\vec{A}$ and $\vec{B}$ as in the previous problem, and $\vec{C} = 5\vec{\imath} + 3\vec{\jmath}$
   (a) find $\vec{A} \times \vec{B}$, and
   (b) determine whether $\vec{A} \times \vec{B}$ is perpendicular to $\vec{C}$.

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3. (22) Given these points: A:(2,1,1), B:(1,2,1), C:(1,1,2).
   (a) Find the area of the triangle determined by the points.
   (b) Find the equation of a plane containing these same points.

6/32
HARDEST

4. (14) Which (if any) of these three lines are parallel? (Give some reasons for your answer.)

$$L_1: \quad \frac{x+3}{2} = \frac{2-y}{2} = \frac{z+6}{5}$$
$$L_2: \quad \frac{x-3}{6} = \frac{y+3}{6} = \frac{2-z}{15}$$
$$L_3: \quad \frac{x-9}{4} = \frac{y-1}{4} = \frac{z+5}{-10}$$

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EASIEST

5. (12) What do the three standard cross-sections of the quadric surface $x^2 = 2y^2 - z$ look like?

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6. (16) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$.

STATS

HI 100 MED ⟨75.5⟩ σ 18.32
LO 42 MEAN 73.44

Mt 13 - F24 - MID III

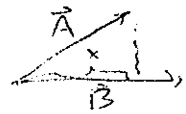
1. $\vec{A} = 3\vec{i} - 2\vec{j} + 4\vec{k}$ $\quad \vec{B} = 2\vec{i} + 5\vec{j} - 4\vec{k}$

a) $\vec{A}\cdot\vec{B} = 3\cdot 2 - 2\cdot 5 - 4\cdot 4 = 6 - 10 - 16 = -20$

$|\vec{A}| = \sqrt{9+4+16} = \sqrt{29}$ $\quad |\vec{B}| = \sqrt{4+25+16} = \sqrt{45}$

$\theta = \arccos\left(\frac{-20}{\sqrt{29}\sqrt{45}}\right)$

b)



$\cos\theta = \frac{x}{|\vec{A}|} \Rightarrow x = (\cos\theta)|\vec{A}| = \frac{-20}{\sqrt{29}\sqrt{45}}\cdot\sqrt{29} = \underline{\frac{-20}{\sqrt{45}}}$

2. a) $\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 4 \\ 2 & 5 & -4 \end{vmatrix} = \vec{i}\,8 + \vec{j}\,8 + \vec{k}\,15 + 4\vec{k} - 20\vec{i} + 12\vec{j}$
$= \underline{-12\vec{i} + 20\vec{j} + 19\vec{k}}$

b) $\vec{C} = 5\vec{i} + 3\vec{j} \Rightarrow (\vec{A}\times\vec{B})\cdot\vec{C} = \langle -12, 20, 19\rangle\cdot\langle 5, 3, 0\rangle$
$= -60 + 60 + 0 = \underline{0} \quad \therefore \perp$

3. $A:(2,1,1)$ $\quad B:(1,2,1)$ $\quad C(1,1,2)$

a) $\vec{AB} = -\vec{i} + \vec{j}$, $\vec{AC} = -\vec{i} + \vec{k}$

Area of $\Delta = \frac{|\vec{AB}\times\vec{AC}|}{2} = \frac{1}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$
$= \frac{1}{2}\left|\vec{i} + \vec{j} + \vec{k}\right| = \underline{\frac{\sqrt{3}}{2}}$

b) $\vec{N} = \vec{AB}\times\vec{AC} = \vec{i} + \vec{j} + \vec{k}$
$\Rightarrow 1(x-2) + 1(y-1) + 1(z-1) = 0 \Rightarrow x + y + z = 2 + 1 + 1 \Rightarrow \underline{x + y + z = 4}$

4. $L_1: \frac{x+3}{2} = \frac{2-y}{2} = \frac{z+6}{5} \parallel 2\vec{i} - 2\vec{j} + 5\vec{k}$

$L_2: \frac{x-3}{6} = \frac{y+3}{6} = \frac{2-z}{15} \parallel 6\vec{i} + 6\vec{j} - 15\vec{k} \parallel 2\vec{i} + 2\vec{j} - 5\vec{k}$

$L_3: \frac{x-9}{4} = \frac{y-1}{4} = \frac{z+5}{-10} \parallel 4\vec{i} + 4\vec{j} - 10\vec{k} \parallel 2\vec{i} + 2\vec{j} - 5\vec{k}$

$\parallel \Rightarrow L_2$ & L_3 are $\parallel$ since both are $\parallel$ to same vector

5. $x^2 = 2y^2 - z \Rightarrow$ x-const $\Rightarrow c = 2y^2 - z \Rightarrow$ parabola
y-const $\Rightarrow x^2 = c - z \Rightarrow$ parabola
z-const $\Rightarrow x^2 = 2y^2 - c \Rightarrow x^2 - 2y^2 = -c \Rightarrow$ hyperbola

6. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1} - 1} = \lim \frac{x^2+y^2}{\sqrt{x^2+y^2+1} - 1}\cdot\frac{\sqrt{x^2+y^2+1} + 1}{\sqrt{x^2+y^2+1} + 1}$

$= \lim \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+1} + 1\right)}{x^2+y^2+1 - 1} = \lim \sqrt{x^2+y^2+1} + 1 = \sqrt{1} + 1 = \underline{2}$