Math 13 (sect. 3) (10:30 AM)—Fall, 2024
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Midterm II (45 minutes)
Friday, October 25, 2024

NOTES:

1. Make sure your NAME is on the front of the green book.
2. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
3. PLEASE *label* (or underline or box in) all ANSWERS clearly.
4. There are 100 points possible on this test. The point value of each problem is listed in parentheses after the number.
5. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
6. **NO CALCULATORS!**

perfect scores per problem

28/32 "EASIEST"

1. (14) Find the line tangent to the curve given parametrically by $x = 2t$ and $y = t^2 + 1$ when $t = 2$.

17/32

2. (14) Compute the length of the curve given parametrically by $x = t^2/2$ and $y = (2t+1)^{3/2}/3$ between the points where $t = 0$ and $t = 4$.

10/32 "HARDEST"

3. (14) Find the Cartesian equation of the curve that has the following parametric equations: $x = \tan t$ and $y = \sin t$.

13/32

4. (28) Identify the following curves (if they exist):
   (a) $2x^2 - 3y^2 + 4x - 12 = 0$
   (b) $y^2 + 2y - x + 2 = 0$
   (c) $x^2 + y^2 - 4x + 2y + 10 = 0$
   (d) $3x^2 + 4y^2 - 6x = 8$

27/32

5. (18) Let $\vec{A} = \vec{\imath} - 2\vec{\jmath} + 6\vec{k}$ and $\vec{B} = 4\vec{\jmath} - 5\vec{k}$.
   (a) Find the directions of $\vec{A}$ and $\vec{B}$.
   (b) Are $\vec{A}$ and $\vec{B}$ the same length?

12/32

6. (12) Let $\vec{A} = \vec{\imath} - 3\vec{\jmath} + 3\vec{k}$, $\vec{B} = -\vec{\imath} + 2\vec{\jmath} + 5\vec{k}$, and $\vec{C} = 2\vec{\jmath} - 16\vec{k}$, determine whether vector $\vec{D} = \vec{A} + \vec{B}$ is parallel to $\vec{C}$. *Give reasons for your answer.*

STATS

(H) 100
LO 44

MED 88
MEAN 86.6

σ 10.97

M+ 13 -F 24 - MID II 10:30 AM

1. $x = 2t \quad y = t^2+1 \Rightarrow \frac{dx}{dt} = 2 \quad \frac{dy}{dt} = 2t \Rightarrow \frac{dy}{dx} = \frac{2t}{2} = t\Big|_{t=2} = 2$

$x|_{t=2} = 4 \quad y|_{t=2} = 5$

$\Rightarrow 2 = \frac{y-5}{x-4} \Rightarrow y-5 = 2x-8 \Rightarrow y = 2x-3$

2. $x = \frac{t^2}{2} \quad y = \frac{(2t+1)^{3/2}}{3}$

$\frac{dx}{dt} = \frac{2t}{2} = t \quad \frac{dy}{dt} = \frac{3}{2}\cdot\frac{(2t+1)^{1/2}}{3}\cdot 2 = (2t+1)^{1/2}$

$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^2 + 2t + 1 = (t+1)^2$

$L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int \sqrt{(t+1)^2}\,dt$

$= \int t+1\,dt = \frac{t^2}{2} + t\Big]_0^4$

$= \frac{16}{2} + 4 - 0 = 8+4 = \underline{12}$

3. $x = \tan t \quad y = \sin t$

$= \frac{\sin t}{\cos t}$

$\Rightarrow x = \frac{y}{\cos t} \Rightarrow \cos t = \frac{y}{x}$

Since $\sin^2 t + \cos^2 t = 1$

$y^2 + \frac{y^2}{x^2} = 1$

4. a) $2x^2 - 3y^2 + 4x - 12 = 0$

$2(x^2+2x+1) - 3y^2 = 12+2 = 14$

$\frac{(x+1)^2}{7} - \frac{y^2}{14/3} = 1$ hyperbola

b) $y^2 + 2y - x + 2 = 0$

$y^2 + 2y + 1 = x - 2 + 1$

$(y+1)^2 = x - 1$

$(y+1)^2 + 1 = x$ parabola

c) $x^2 + y^2 - 4x + 2y + 10 = 0$

$(x^2-4x+4) + (y^2+2y+1) = -10+4+1$

$(x-2)^2 + (y-1)^2 = -5$

impossible

d) $3x^2 + 4y^2 - 6x = 8$

$3(x^2-2x+1) + 4y^2 = 8+3 = 11$

$\frac{(x-1)^2}{11/3} + \frac{y^2}{11/4} = 1$ ellipse

5. $\vec{A} = \vec{i} - 2\vec{j} + 6\vec{k} \qquad \vec{B} = 4\vec{j} - 5\vec{k}$

a) $\hat{u}(\vec{A}) = \frac{\vec{i}-2\vec{j}+6\vec{k}}{\sqrt{1+4+36}} = \frac{\vec{i}-2\vec{j}+6\vec{k}}{\sqrt{41}} \qquad \hat{u}(\vec{B}) = \frac{4\vec{j}-5\vec{k}}{\sqrt{16+25}} = \frac{4\vec{j}-5\vec{k}}{\sqrt{41}}$

b) yes, both $\vec{A}$ and $\vec{B}$ have length $\sqrt{41}$

6. $\vec{A} = \vec{i} - 3\vec{j} + 3\vec{k} \qquad B = -\vec{i} + 2\vec{j} + 5\vec{k} \qquad \vec{C} = 2\vec{j} - 16\vec{k}$

$\Rightarrow \vec{D} = -\vec{j} + 8\vec{k}$

$\vec{C} = (-2)\vec{D}$

$\therefore \vec{C} \parallel \vec{D}$