Math 13-3 (10:30 AM)—Fall, 2024 Midterm I (45 minutes)
D.C. Smolarski, S.J. Friday, October 11, 2024

NOTES:

1. Make sure your NAME is on the front of the green book.
2. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
3. PLEASE *label* (or underline or box in) all ANSWERS clearly.
4. There are 100 points possible on this test. The point value of each problem is listed in parentheses after the number.
5. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
6. **NO CALCULATORS!**

perfect scores per problem

14/33 1. (10) Find $\frac{dy}{dx}$ if $\arcsin y = \sinh x$. The answer should be all in terms of x.

12/33 "HARDEST" 2. (12) Find the area between the curve $y = -\sinh x + 3$ and the x-axis between $x = -1$ and $x = 1$. Reduce the answer *fully*, i.e., eliminate from the answer any hyperbolic trig functions.

24/33 3. (12) Given the sequence 2/7, 4/8, 6/9, 8/10, ...,

(a) find an expression for the general term a_n.

(b) determine whether the sequence converges or not, and, if so, find its limit.

19/33 4. (12) Determine whether $\sum_{n=3}^{\infty} \frac{4}{5^n}$ converges or diverges. If it converges, determine the sum.

25/33 "EASIEST" 5. (12) Find the sum of $\sum_{n=2}^{\infty} \frac{1}{n^2 + 3n + 2}$.

19/33 6. (12) Create the Maclaurin polynomial of degree 2 for $f(x) = \sin^2 3x$.

16/33 7. (15) Create the Taylor polynomial of degree 2 for $f(x) = 2xe^x$ around the expansion point $a = 2$.

19/33 8. (15) The Maclaurin series for $e^x = 1 + x + x^2/2! + x^3/3! + \cdots$. *Using this series,* compute

$$\lim_{x \to 0} \frac{2e^x - 2e^{-x}}{x}.$$

STATS

HI 100 MED 84 σ 19.14

LO 36 MEAN 79.03

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1. $\arcsin y = \sinh x = \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \cosh x \Rightarrow \frac{dy}{dx} = \sqrt{1-y^2}\cosh x$

$\Rightarrow \frac{dy}{dx} = \cosh x \sqrt{1-\sin^2(\sinh x)} = (\cosh x)\sqrt{\cos^2(\sinh x)} = \cosh x \cdot \cos(\sinh x)$

2. $\int_{-1}^{1} -\sinh x + 3\,dx = -\cosh x + 3x \Big|_{-1}^{1} = -\cosh 1 + 3 - (-\cos(-1) - 3)$

$= -\frac{e^1+e^{-1}}{2} + 3 + \frac{e^{-1}+e^1}{2} + 3 = \underline{6}$

3. 2/7, 4/8, 6/9, 5/10 ..

a) $\frac{2n}{n+6}$ b) $\lim \frac{2n}{n+6} \overset{L'H}{=} \lim \frac{2}{1} = \underline{2}$

4. $\sum_{n=3}^{\infty} \frac{4}{5^n} = \frac{4}{5^3} + \frac{4}{5^4} + \frac{4}{5^5} + \cdots = \frac{4/5^3}{1-1/5} = \frac{4}{5^3}\cdot\frac{5}{4} = \underline{\frac{1}{25}}$

5. $\sum_{n=2}^{\infty} \frac{1}{n^2+3n+2} = \sum \frac{1}{(n+1)(n+2)}$

$= \sum_{2}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

$= \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + \cdots = \underline{\frac{1}{3}}$

$\frac{1}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$

$\Rightarrow 1 = A(n+2) + B(n+1) = An + Bn + 2A + B$

$\Rightarrow 0 = A + B$, $1 = 2A + B$ $\Big\}$ $1 - A \Rightarrow B = -1$

$0 = -A \cdot B$

6. $f(x) = \sin^2 3x$

$f'(x) = 6\sin 3x \cos 3x$

$f''(x) = 3\cdot 6 \sin 3x(-\sin 3x) + 3\cdot 6 \cos 3x \cos 3x$

$= 18(\cos^2 3x - \sin^2 3x)$

$f(0) = 0$

$f'(0) = 0$

$f''(0) = 18$

$\therefore f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2}$

$= 0 + 0 + \frac{18x^2}{2}$

$= \underline{9x^2}$

7. $f(x) = 2xe^x$

$f'(x) = 2xe^x + 2e^x$

$f''(x) = 2xe^x + 2e^x + 2e^x$

$f(2) = 4e^2$

$f'(2) = 4e^2 + 2e^2 = 6e^2$

$f''(2) = 4e^2 + 4e^2 = 8e^2$

$\therefore f(x) = f(2) + f'(2)(x-2) + f''(2)\frac{(x-2)^2}{2}$

$= 4e^2 + 6e^2(x-2) + 8e^2(x-2)^2$

$= \underline{4e^2 + 6e^2(x-2) + 4e^2(x-2)^2}$

8. $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots \Rightarrow e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \cdots$

$\Rightarrow \lim_{x\to 0} \frac{2e^x - 2e^{-x}}{x} = \lim \frac{2}{x}\left[1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots - \left(1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \cdots\right)\right]$

$= \lim \frac{2}{x}\left[2x + \frac{2x^3}{3!} + \cdots\right] = 2\cdot 2 = \underline{4}$