DEF INT + SUBSTITUTION (5.6) p344

If we use substitution to evaluate a definite integral,
we have 2 choices with regard to the limits of integration.

① after integrating, we re-substitute + evaluate using original limits.

② we derive new limits, based on our substitution.
THEN, after integrating, we evaluate immediately using the new limits and WITHOUT re-substitution. ⇒ Thm 7, p344

cf EX ~~#~~1 p. ~~2??~~ 345

$$\int_0^2 \frac{6x^2\,dx}{\sqrt{2x^3+9}}$$

①
$$u = 2x^3+9$$
$$du = 6x^2\,dx$$

$$= \int_{x=0}^{x=2} u^{-\frac{1}{2}}\,du = \left[\frac{u^{\frac{1}{2}}}{1/2}\right]_{x=0}^{x=2} \quad \text{now resubstitute.}$$

$$= 2\sqrt{2x^3+9}\,\Big]_{x=0}^{x=2} = 2\sqrt{2\cdot 2^3+9} - 2\sqrt{2\cdot 0^3+9}$$

$$= 2\sqrt{16+9} - 2\sqrt{9} = 2\sqrt{25} - 2\sqrt{9} = 2\cdot 5 - 2\cdot 3 = 10-6 = \underline{\underline{4}}$$

②
$$u = 2x^3+9 \qquad \Longrightarrow \qquad \text{for } x=2 \quad u = 2\cdot 2^3+9 = 2\cdot 8+9 = 16+9 = \underline{\underline{25}}$$
$$du = 6x^2\,dx \qquad\qquad \text{for } x=0 \quad u = 2\cdot 0+9 = \underline{\underline{9}}$$

$$= \int_{u=9}^{u=25} u^{-\frac{1}{2}}\,du = \left[\frac{u^{\frac{1}{2}}}{1/2}\right]_{u=9}^{u=25} \quad \underline{\underline{\text{NO}}}\ \text{re-subst.}$$

$$= 2\sqrt{u}\,\Big]_{u=9}^{u=25} = 2\sqrt{25} - 2\sqrt{9} = 2\cdot 5 - 2\cdot 3 = 10-6 = \underline{\underline{4}}$$

5.6

APPLICATIONS OF THE DEFINITE INTEGRAL Chpt 5–6

In Chpt 5.3, we essentially developed the definite integral from the area under a curve, and later identified the integral with the limit of a sum.

Using this background, we now look at common applications of the integral:

① change in position
② distance traveled
③ area between 2 curves → chpt 5
④ volumes. → chp 6

Procedure

5.6

① want measurement (eg area)

→ problem, figure is irregular

② subdivide (strips)

③ approximate by regular figure (rectangle)

$(A = \underset{f(x)}{l} \cdot \underset{\Delta x}{w})$

④ sum up to get total <u>approx</u> area $\sum f(x)\Delta x$

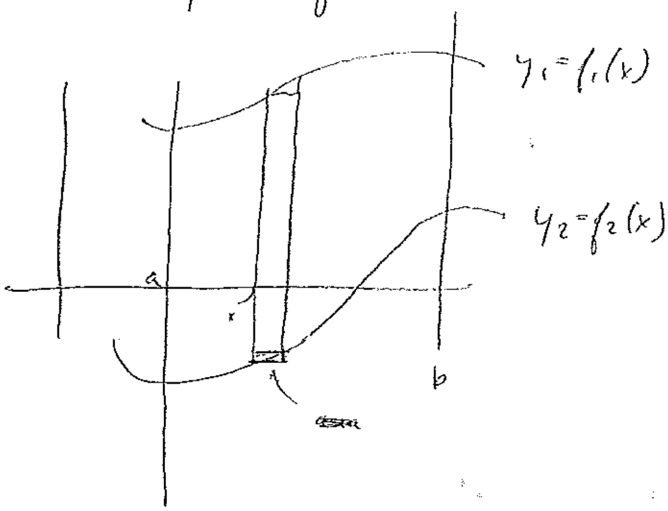
⑤ take limit & convert to integral

$$\left[\int_a^b f(x)\,dx \right]$$

B. AREA BETWEEN 2 CURVES ~~[5-2]~~ ~~p 309~~ (~~5.4~~? 5.6 p347)

Suppose you have 2 functions, $y_1 = f_1(x)$, $y_2 = f_2(x)$. Both are continuous on a closed interval $[a, b]$, and also suppose $f_1(x) \geq f_2(x) \quad \forall x \in [a, b]$.

Let try to find the area between the 2 curves and write it in terms of an integral.



height $= f_1(x) - f_2(x)$
width $= \Delta x$
$\therefore$ area of 1 rectangle $= \left[f_1(x) - f_2(x)\right] \Delta x$

Adding up all rectangles.

$$A_{R,n} = \sum_{a,\; n}^{b} [f_1(x) - f_2(x)]\, \Delta x$$

$$A = \lim_{n \to \infty} A_{R,n} = \lim_{\Delta x \to 0} \sum_{a,\; n}^{b} [f_1(x) - f_2(x)]\, \Delta x$$

$$= \int_a^b [f_1(x) - f_2(x)]\, dx$$

upper curve. lower curve

(p 3~~4~~9) 12^{th} ed

C EXAMPLES

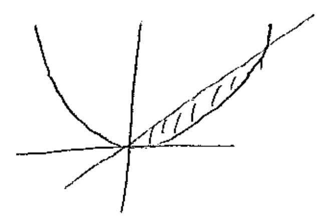
12^{th}

~~B~~ area between curve $y = x^2$ and line $y = x$

p 352 # 66

1st Quest – what are limits of integration on x-axis, i.e. where 2 the 2 lines intersect?

Substitute + solve (i.e. find pts which satisfy both equations simultaneously).



$y = x^2$, $y = x$

$\Rightarrow x^2 = x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$ or $x = 0, 1$

$\therefore y = 0$ or $y = 1$ $\therefore$ pts of intersection are $(0,0)$, $(1,1)$

$\therefore$ we integrate from $x=0$ to $x=1$.

2nd Quest which is upper curve?

$$\int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3} - (0 - 0) = \frac{3}{6} - \frac{2}{6} = \underline{\frac{1}{6}}$$

INTEGRATING W.R.T. Y

If equations are given in terms of x being a function of y, we can just interchange variables in general formulas from x to y, and then use the same formula, since the theory would be the same.

EX p ~~230~~ #3 7th

find area between y-axis and curve $x = y^2 - y^3$

4 EX7 p349

Note: y-axis is line $x = 0$

① pts of intersection?

$$x = 0 = y^2 - y^3$$

$$0 = y^2(1-y) \therefore y = 0 \text{ or } y = 1$$

adapted integral formula is

$$\int_0^1 (y^2 - y^3 - 0)\, dy$$

$$= \frac{y^3}{3} - \frac{y^4}{4} \Big]_0^1 = \frac{1}{3} - \frac{1}{4} - 0 = \frac{1}{12}$$

HW p ~~230~~ 1, 2, 4, 8, 10