(5.1)

PRELIM

Part of Calculus is about finding approximations in order to get desired exact quantity.

Calc I – to find slope of curve = slope of tangent line we found slope of secant lines (approx.) + took limit. (exact)

We will do something similar now.

→ 2, 4, 6, 8

How do we approximate?

Paper Demo

→ rect

→ circle

→ curve & cut.

→ Need some NOTATION (Σ)

SUMMATION SYMBOL (5,2) p 307

$\sum$ is capital Greek s = sigma

It is used to indicate a repeated sum.

eg. $\sum_{j=1}^{5} j = 1+2+3+4+5$

or $\sum_{j=1}^{3} (j^2+2) = (1^2+2)+(2^2+2)+(3^2+3)$

In general

$$\sum_{j=1}^{K} f(j) = f(1)+f(2)+f(3)+\cdots+f(K-1)+f(K)$$

increments of 1

(5.8) 4

of pg 312

#2 #16. Write out $\sum_{k=1}^{5} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

#20 #14. Find value $\sum_{n=0}^{4} \frac{n}{4} = \frac{0}{4} + \frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4} = \frac{10}{4} = \frac{5}{2}$

13)

SUMMATION SYMBOL PRACTICE

How much is $\sum_{i=1}^{12}\left(\sum_{j=i}^{12} i\right)$?

(5.2)

A SUMMATION FORMULAE ~~(4.6)~~ ~~p 277~~ p 309

(89-5)

To use the definition of the Riemann integral for computing areas we first need a few summation formulae, which can be proven by means of mathematical induction cf ~~Ap. 2~~ pp A-6 ff

A. 2 p AP-6 7+

[5 ~~A-21 ff~~] 8

(EX 4 p 309)

Thm 1 $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad (= 1+2+3+\dots+n)$$

proof

step 1 Show true for $n=1$

$$\sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2} = \frac{1\cdot 2}{2} = 1 \quad \text{OK.}$$

step 2 assume true for $n=p$, i.e.

$$\sum_{k=1}^{p} k = \frac{p(p+1)}{2}$$

Then add $p+1$ to the left side:

$$\left(\sum_{k=1}^{p} k\right) + (p+1) = \sum_{k=1}^{p+1} k = \frac{p(p+1)}{2} + (p+1)$$

$$= \frac{p(p+1)}{2} + \frac{2(p+1)}{2} = \frac{(p+1)(p+2)}{2}$$

$$= \frac{(p+1)[(p+1)+1]}{2}$$

// QED.

Overhead + Computer

Formula p 309

wait until end? step?

p. 278

p. 273

Thm 2 $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \left(= 1^2 + 2^2 + 3^2 + \cdots + n^2\right)$

proof step 1 $1 = \sum_{k=1}^{1} k^2 = \frac{1(1+1)(2\cdot1+1)}{6} = \frac{1\cdot2\cdot3}{6} = 1$ ok.

step 2 assume true for $n = p$, i.e.

$$\sum_{k=1}^{p} k^2 = \frac{p(p+1)(2p+1)}{6}$$

add $(p+1)^2$ to the left side.

$$\sum_{k=1}^{p+1} k^2 = \frac{p(p+1)(2p+1)}{6} + (p+1)^2$$

$$= \frac{p(p+1)(2p+1) + 6(p+1)^2}{6}$$

$$= \frac{(p+1)}{6}\left[(2p^2+p) + (6p+6)\right]$$

$$= \frac{(p+1)}{6}\left(2p^2+7p+6\right)$$

$$= \frac{(p+1)}{6}(2p+3)(p+2)$$

$$= \frac{(p+1)((p+1)+1)(2(p+1)+1)}{6}$$

// QED

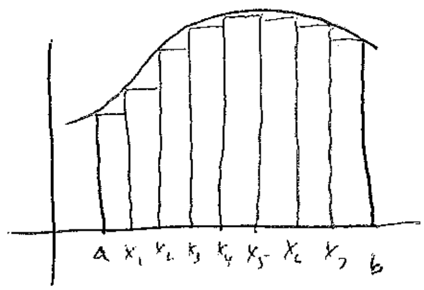
(89-4) 1-1

AREA UNDER A CURVE

(4,5) 7
(4,3) 25

p 311-12 (5,2)

- In Chpt 1, ~~[illegible]~~ to obtain the slope of a curve at a point, we started by approximating the slope by the slope of various secant lines and then looking at the limit.
- We will now use a similar process to find the area between a curve and the x-axis.
- Given a curve, given 2 end points a and b, we start approximating the area by building rectangles under the curve.



- The approximate area under the curve is the sum of the areas of the <u>inscribed</u> rectangles.
- The area of one rectangle is w·h. Assume all base widths are equal and $= \Delta x$. The height is $f(x_i)$ for the correct x_i ∴ The area of a rectangle is $f(x_i)\,\Delta x$
- The total area of all the rectangles is $\sum f(x_i)\,\Delta x$ (if x_i is chosen correctly).

  where $\Delta x = \frac{b-a}{n}$

called a Riemann Sum

(p 312)

- If we increase the number of rectangles, we get a better approximation. But this means the width of each rectangle gets smaller, i.e. Δx gets smaller.

- The actual area is defined as a limit. ~~p356~~

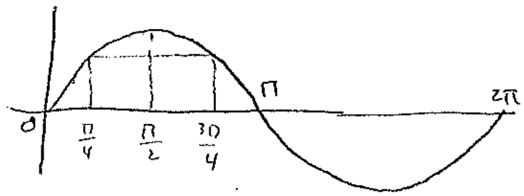
$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(c_k) \Delta x \qquad p\,314 \quad (5.3)$$

where $f(c_k)$ is the smallest value of f on the interval $[x_{k-1}, x_k]$.

EXAMPLES

|| Math 170 "4B"
p. 325
#45

$y = \sin x$ let $n = 4$ be the number of rectangle
let $a = 0$, $b = \pi$
Find approx area.



$\Delta x = \dfrac{b-a}{n} = \dfrac{\pi - 0}{4} = \dfrac{\pi}{4}$

1st rect height $= 0$
2nd " " $= \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
3rd " " $= \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$
4th " " $= 0$

approx area $= 0 \cdot \frac{\pi}{4} + \frac{\sqrt{2}}{2} \cdot \frac{\pi}{4} + \frac{\sqrt{2}}{2} \cdot \frac{\pi}{4} + 0 \cdot \frac{\pi}{4}$

$= 2\left(\frac{\sqrt{2} \cdot \pi}{8}\right) = \frac{\pi\sqrt{2}}{4}$

(5.2)