ANTIDERIVATIVES (4.8) p279

In general, antidifferentiation or integration is the reverse of the process of finding derivatives. That is, given a function f, we want to find which function F it is that f is the derivative of.

F is called an antiderivative of f, i.e. $F'(x)=f(x)$.

The set of all antiderivatives is called the integral of f.

In particular, there are 2 types of integration, definite and indefinite. We will look at indefinite first. Definite integration will be discussed later.

DIFFERENTIAL EQUATIONS

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Given an equation of the form

$$\frac{dy}{dx} = f(x)$$

frequently we want to know what function $f(x)$ came from, i.e. we want $y = F(x)$ s.t. $F'(x) = f(x) = \frac{dy}{dx}$.

This type of equation is called a <u>differential equation</u> or DIFF-E-Q.

The function $F(x)$ is called the <u>solution</u> of the Diff EQ iff $F(x)$ is diff'ble and $\frac{dF(x)}{dx} = f(x)$.

we can also call $F(x)$ the <u>antiderivative</u> or <u>primitive</u> of $f(x)$

INTEGRALS (p 284)

We indicate the process of integration, i.e. of finding the ~~(solution to a diff eq.)~~ Set of all antiderivatives by means of an integral sign which is a stylized S. standing for sum. (more on sum later)

$$\int \ldots dx$$

i.e. $$\int f(x)\,dx = F(x)$$

NOTE: Think of $\int$ as half of a matched pair – the other half is the dx or dy or dt. This tells us the variable of integration. NEVER have an integral sign w/o a dx or something at the end – everything else is between, like parentheses.

Thm Let $F_1(x) = F_2(x) + C$ for any constant C. cf Cor 2 p233 Thm 8 p280

Let $F_2'(x) = f(x)$.

Then 1) $F_1'(x) = f(x)$

and 2) $\int f(x)\,dx = F_2(x) + C = F_1(x)$.

NOTE: The importance of this Thm is to point out that Integration is accurate only up to a constant!

I'll sometimes indicate C_5 to emphasize you may get 5 pts off if you forget to write the constant.

STANDARD FORMULAE

1) $\int dx = x + C_5$

or $\int \frac{du}{dx}\, dx = u(x) + C_5$

2). $\int a\, u(x)\, dx = a \int u(x)\, dx$ a a constant

3) $\int u(x) + v(x)\, dx = \int u(x)\, dx + \int v(x)\, dx$

4) $\int u^n \frac{du}{dx}\, dx = \frac{u^{n+1}}{n+1} + C_5$

5) $\int \sin Kx\,dx = -\frac{1}{K}\cos Kx + C$

6) $\int \cos Kx\,dx = +\frac{1}{K}\sin Kx + C$

7) $\int \sec^2 Kx\,dx = \frac{1}{K}\tan Kx + C$

8) $\int \sec Kx \tan Kx\,dx = \frac{1}{K}\sec Kx + C$

9) $\int e^{Kx}\,dx = \frac{1}{K}e^{Kx} + C$

10) $\int \frac{1}{x}\,dx = \ln|x| + C$

11) $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

12) $\int \frac{dx}{1+x^2} = \arctan x + C$

13) $\int \frac{dx}{x\sqrt{x^2-1}} = \operatorname{arcsec} x + C$

$\leftarrow$

EXAMPLES

$$\int 2x+3\,dx = 2\int x\,dx + 3\int dx$$

$$= 2\frac{x^2}{2} + 3x + C_1 + C_2$$

$$= x^2 + 3x + C$$

Is this really correct?

Check by differentiating.

$$\frac{d}{dx}\left(x^2+3x+C\right) = 2x+3+0 = \underline{2x+3}$$

EVALUATING THE CONSTANT

I – Checking for pts on a curve.

Is $(1,2)$ on $y = x^2+1$?

Is $(1,4)$ on $y = 3x-2$?

II – "Families" of curves with identical slope.

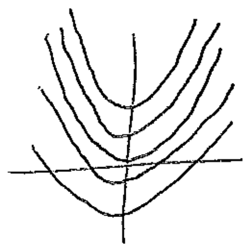
$y = x^2$

$y = x^2+1$

$y = x^2+2$

$y = x^2-1$

$y = x^2-3$



which one goes through $(1,3)$?

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EX # 93
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$\frac{dy}{dx} = \frac{1}{x^2} + x \qquad y(2) = 1 \qquad \Rightarrow (2,1)$ is on curve

$\Rightarrow \int dy = \int x^{-2} + x \, dx$

$y = \frac{x^{-1}}{-1} + \frac{x^2}{2} + C$

$= -\frac{1}{x} + \frac{x^2}{2} + C$

Eval at $(2,1)$

$\Rightarrow 1 = -\frac{1}{2} + \frac{2^2}{2} + C \quad = -\frac{1}{2} + 2 + C \; = \frac{3}{2} + C$

$\Rightarrow 1 - \frac{3}{2} = C$

$\Rightarrow -\frac{1}{2} = C$

$\therefore$ sol of diff eq is.

$y = -\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$

APPLICATIONS

We would like to evaluate the arbitrary constant C which appears as a result of integration. If we are given some additional conditions on a diff. eq., most often we can determine the constant C.

EXE 2

$vel = \frac{ds}{dt} = at$ with a constant

If $s = s_0$ when $t = 0$ find s as a function of t.

$$v = \frac{ds}{dt} \Rightarrow ds = at\,dt$$

$$\Rightarrow \int ds = \int at\,dt$$

$$\Rightarrow s = a\frac{t^2}{2} + C$$

But at $t=0$, $s = s_0$ $\therefore$ $s_0 = a \cdot 0 + C$

$$\therefore C = s_0$$

i.e. $s = a\frac{t^2}{2} + s_0$

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202-3

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q. ex5

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$a = 6t \quad v_0 = 0 \quad s_0 = 5$

$a = 6t$

$\dfrac{dv}{dt} = 6t \Rightarrow dv = 6t\,dt$

$\Rightarrow v = \int 6t\,dt = \dfrac{6t^2}{2} + C_1$

$= 3t^2 + C_1$

$0 = 3 \cdot 0 + C_1 \quad \therefore C_1 = 0$

$\therefore v = 3t^2$

$\dfrac{ds}{dt} = 3t^2 \Rightarrow ds = 3t^2\,dt$

$s = \int 3t^2\,dt = t^3 + C_2$

$5 = 0 + C_2 \quad \therefore C_2 = 5$

$\therefore \quad \underline{\underline{s = t^3 + 5}}$