Non-Separable Diff EQ

Ex2 p2

$$\frac{dy}{dt} = y + t$$

$y = Ce^t - (t+1)$ is a sol. Let's check

$$\frac{dy}{dt} = Ce^t - 1$$

$$Ce^t - 1 = Ce^t - (t+1) + t$$

$$= Ce^t \cancel{-t} - 1 + \cancel{t}$$

$$= Ce^t - 1$$

Yes!

Given $\frac{dy}{dt} = te^{y}$ with $y=3$ when $t=6$

usually we integrate first & then substitute the initial conditions to evaluate the $\underline{C}$. (ans: $y = -\ln\left(-\frac{1}{2}t^2+18+\frac{1}{e^3}\right)$)

Alternatively we can use definite integration.

$$\int_3^{y(t_f)} e^{-y}\,dy = \int_6^{t_f} t\,dt$$

$$\Rightarrow \quad -e^{-y}\Big|_3^{y(t_f)} = \frac{t^2}{2}\Big|_6^{t_f}$$

$$\Rightarrow \quad -e^{-y(t_f)} + e^{-3} = \frac{t_f^2}{2} - \frac{36}{2} = \frac{t_f^2}{2} - 18$$

$$\Rightarrow \quad e^{-y(t_f)} = \frac{-t_f^2}{2} + 18 + \frac{1}{e^3}$$

$$\Rightarrow \quad \ln\left(e^{-y(t_f)}\right) = \ln\left(-\frac{t_f^2}{2} + 18 + \frac{1}{e^3}\right)$$

$$-y(t_f) = \ln\left(-\frac{t_f^2}{2} + 18 + \frac{1}{e^3}\right)$$

$$y(t) = -\ln\left(-\frac{t^2}{2} + 18 + \frac{1}{e^3}\right)$$

EXAMPLE

EX 7

pg 5-6

$$PV = nRT \quad \Rightarrow V = \frac{nRT}{P}$$

"ideal gas"

$$P\frac{dV}{dt} + V\frac{dP}{dt} = 0$$

$$\frac{dV}{dt} = -\frac{V}{P}\frac{dP}{dt}$$

$$= -\frac{nRT}{P^2}\frac{dP}{dt}$$

Given $\frac{dV}{dt} = -t^3$ and $P(0)=3$

$$-t^3 = \frac{-nRT}{P^2}\frac{dP}{dt}$$

$$-t^3\,dt = -nRT\,P^{-2}\,dP$$

$$\int_0^{t_f} t^3\,dt = nRT\int_3^{P(t_f)} P^{-2}\,dP$$

$$\left.\frac{t^4}{4}\right]_0^{t_f} = nRT\left(-P^{-1}\right)\Big|_3^{P(t_f)}$$

$$\frac{t_f^4}{4} = nRT\left[-\frac{1}{P(t_f)} + \frac{1}{3}\right] = \frac{nRT}{3} - \frac{nRT}{P(t_f)}$$

$$\Rightarrow \frac{nRT}{P(t_f)} = \frac{nRT}{3} - \frac{t_f^4}{4}$$

$$\Rightarrow \frac{1}{P(t_f)} = \frac{1}{3} - \frac{t_f^4}{4nRT}$$

$$\Rightarrow P(t) = \frac{1}{\frac{1}{3} - \frac{t^4}{4nRT}}$$