92-15 18-1

88-18

SIGN OF FIRST DERIVATION (~~23~~)(43)

The derivative tells us the slope of a given function.

Cal 3 p 238

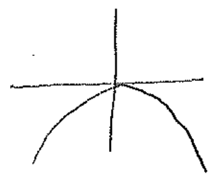
If the slope is + at a point, the function is <u>rising</u> (increasing)

if −, the function is <u>falling</u>. (decreasing)

This information can be very useful when graphing functions.

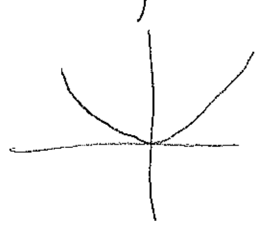
<u>Note</u>: The sign of the derivative is unrelated to the sign of a function itself.

E.G. ① Let $y = -x^2$



$\Rightarrow$ for $x < 0$ the deriv is pos even though y is always neg.

② Let $y = x^2$



for $x < 0$ the deriv is neg. even though y is always pos.

<u>NOTE</u>: If slope is 0, the curve must have a horizontal tangent.

By finding the points of zero slope, we can subdivide a curve into sub-intervals in which the curve is either rising or falling. These points (along with points where there is no slope) are called "critical pts".

cf. Sal 4.1 p 226

EG.

~~p185 #9~~

NOT in

88 E1.

12

cf EX 1

p239

Given

$$y = x^2 - x + 1$$

where is it rising and falling?

$$\frac{dy}{dx} = 2x - 1$$

if $\frac{dy}{dx} = 0$, then $2x - 1 = 0$

or $x = \frac{1}{2}$

This is a point with horizontal tangent

Pick 2 points, one on either side of $\frac{1}{2}$, e.g. 0 and 1.

$$\left.\frac{dy}{dx}\right|_{x=0} = 2 \cdot 0 - 1 = -1 < 0$$

$\Rightarrow$ for $x < \frac{1}{2}$ curve is falling.

$$\left.\frac{dy}{dx}\right|_{x=1} = 2 \cdot 1 - 1 = 2 - 1 = 1 > 0$$

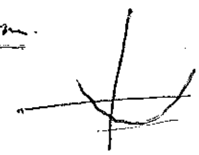
$\Rightarrow$ for $x > \frac{1}{2}$ curve is rising

HORIZONTAL TANGENTS

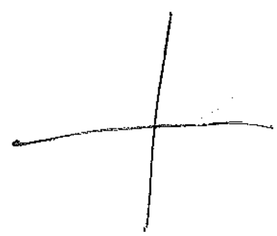
[As mentioned before], if the slope of a curve is 0, the curve must have a horizontal tangent.

If the slope changes from pos to neg as it passes the hor. tang, at the hor. tangent it reaches a local high point or <u>local maximum</u>.

If the slope changes from neg to pos as it passes the hor. tang., at the hor. tang it reaches a local low point or <u>local minimum</u>.



It is also possible for a curve to have a horizontal tangent without either a local min or max. eg. $y=x^3$



These facts can be very useful when graphing a function.

<u>WARNING</u> It <u>is</u> possible to have a function with a min or max and <u>no</u> hor. tangent. eg. $y=|x|$

at $x=0$, y attains a minimum, but no derivative is defined!

CONCAVITY

12 (4.4) p243

— When a curve has a "regular-type" maximum, the slope changes from pos to zero to negative, i.e. it is decreasing. The shape of such a curve segment is said to be concave downward (spills water)

— When a curve has a "regular-type" minimum, the slope changes from neg to zero to positive, i.e. it is increasing. The shape of such a curve segment is said to be concave upward (holds water).

SECOND DERIVATIVE

The 1st derivative tells us the rate of change of the height (y) with respect to x.

The 2nd derivative tells us the rate of change of the 1st deriv (slope) with respect to x.

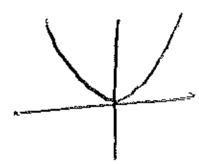
cf Thm 5 p 246

If 2nd deriv > 0, then the 1st deriv is increasing, i.e. the slope is increasing or we have a concave upward shape.

If 2nd deriv < 0, then the 1st deriv is decreasing, i.e. the slope is decreasing or we have a concave downward shape.

EG

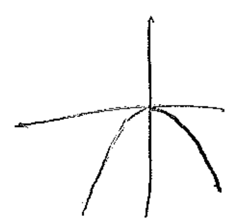
① parabola $y = x^2$



$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2 > 0$$

② parabola $y = -x^2$



$$\frac{dy}{dx} = -2x$$

$$\frac{d^2y}{dx^2} = -2 < 0$$

INFLECTION

v2 p244

If a curve changes concavity at a point, this point is called a "point of inflection".

If 2nd deriv is pos at one point and neg at another, ∃ a good chance that it is 0 at some pt in between.

However, it is possible that y'' will not exist, .

EXAMPLE

$$y = x^3 + 2x^2$$

$$\frac{dy}{dx} = 3x^2 + 4x$$

$$\frac{d^2y}{dx^2} = 6x + 4$$

now if $6x + 4 = 0$

$6x = -4$

$x = -\frac{2}{3}$

$\therefore -\frac{2}{3}$ is a point of inflection

Test Let $x = -1$ $(< -\frac{2}{3})$

$\Rightarrow y''\big|_{x=-1} = -6 + 4 = -2 < 0$ $\therefore$ conc. down

Let $x = 0$

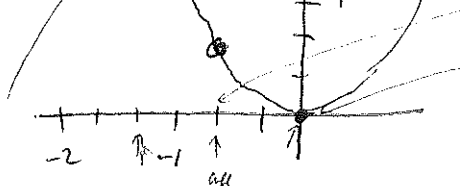
$\Rightarrow y''\big|_{x=0} = 0 + 4 > 0$ $\therefore$ conc. up.

Where is there a level slope?

$y' = 3x^2 + 4x = 0$

$(3x + 4)x = 0 \Rightarrow x = 0$ or $x = -\frac{4}{3}$

Graph



check heights at key points

x	y
$-\frac{2}{3}$	$-\frac{8}{27} + \frac{6}{9} = \frac{-8}{27} + \frac{24}{27} = \frac{16}{27}$
0	0
$-\frac{4}{3}$	$\frac{-64}{27} + \frac{32}{9} = \frac{-64+96}{27} = \frac{32}{27}$