

~~8.12 7.5~~ 89-23  
8.4 p 471

## PARTIAL FRACTIONS

Look at  $\frac{17}{35}$ . In theory it can be divided into the sum of  $\frac{A}{5}$  and  $\frac{B}{7}$ .

$$\frac{17}{35} = \frac{A}{5} + \frac{B}{7} = \frac{7A + 5B}{35}$$

Let  $A=1, B=2$

This is not uniquely determined in this case, <sup>let  $A=6, B=-5$</sup>  But it can be unique in other cases.

Look at  $\frac{5x-3}{x^2-2x-3}$ . We can factor the bottom into  $(x+1)(x-3)$ .

$$\therefore = \frac{5x-3}{(x+1)(x-3)}. \text{ We would like to break this down into}$$

a sum of 2 fractions:  $\frac{A}{x+1} + \frac{B}{x-3}$

$$\frac{A}{x+1} + \frac{B}{x-3} = \frac{5x-3}{(x+1)(x-3)}$$

Cross multiply

$$A(x-3) + B(x+1) = 5x-3$$

$$Ax - 3A + Bx + B = 5x - 3$$

$$(A+B)x + (B-3A) = 5x - 3$$

This is true iff like powers of  $x$  have identical coefficients.

i.e.  $A+B=5$  - coeff of  $x$   
 $-3A+B=3$  - constant term.

Now solve the  $2 \times 2$  linear system.

$$\textcircled{1} - \textcircled{2} \Rightarrow 4A = 8 \Rightarrow A = 2$$

$$\therefore B = 3$$

$$\therefore \frac{5x-3}{x^2-2x-3} = \frac{2}{x+1} + \frac{3}{x-3}$$

$$\therefore \int \frac{5x-3}{x^2-2x-3} dx = 2 \int \frac{dx}{x+1} + 3 \int \frac{dx}{x-3}$$

7<sup>th</sup> of EX1  
p 482  
8.2 508

12 p 471

# ADDITIONAL CASES

(See Box pp 472-73)

#1 (cf EX 2 p ~~472~~<sup>508 or 12 473</sup>) If one of the linear factors in the denominator is raised to a power i.e.  $(x-r)^m$

then you must have m partial fractions on the other side whose denominators range from  $(x-r)$  to  $(x-r)^m$ .

eg.

$$\frac{A_1}{x-r} + \frac{A_2}{(x-r)^2} + \frac{A_3}{(x-r)^3} + \dots + \frac{A_m}{(x-r)^m}$$

#2 (cf EX 4 p ~~483~~<sup>509 474</sup>) If you have a non-factorable quadratic factor

eg.  $x^2 + px + q$

then this may be the sum of 2 partial fractions, one of which has an x in the numerator,

i.e.

$$\frac{A}{x^2 + px + q} + \frac{Bx}{x^2 + px + q}$$

NOTE: In practice, it is easier to use  $\frac{A}{x^2 + px + q} + \frac{Bx + C}{x^2 + px + q}$

#3 EX 5 p 475 If a quadratic factor is raised to a power, combine procedures in cases #1 and #2.

eg.  $(x^2 + px + q)^m$  corresponds to

$$\frac{A_1 + B_1x}{x^2 + px + q} + \frac{A_2 + B_2x}{(x^2 + px + q)^2} + \dots$$

(cf EX 3 p 474)

#4 If the rational fraction  $\frac{f(x)}{g(x)}$  is such that  $\deg f(x) \geq \deg g(x)$ , then divide first and then obtain  $h(x) + \frac{f_2(x)}{g(x)}$  where  $\deg f_2(x) < \deg g(x)$ .

The factors of  $g(x)$  should be known. In theory, any polynomial  $g(x)$  with real coefficients can be expressed as a product of real linear + quadratic factors. In practice, it may be difficult to factor.

72  
cf. Ex 6 p 484  
→ 472

MORE EXAMPLES

NOTE: If you have a quadratic factor in the denominator, it is often easier to use its derivative as a factor in the numerator, rather than a plain  $x$ .

EX 72  
~~S/2~~  
~~#38~~  
~~A 489~~  
~~#26~~

$$\int_0^1 \frac{dx}{(x+1)(x^2+1)}$$

deriv. of bottom

$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{B(2x)}{x^2+1} + \frac{C}{x^2+1}$$

$$1 = A(x^2+1) + B(2x)(x+1) + C(x+1)$$

$$= Ax^2 + A + 2Bx^2 + 2Bx + Cx + C$$

$$x^2: 0 = A + 2B$$

$$x: 0 = 2B + C$$

$$c: 1 = A + C$$

$$\left. \begin{array}{l} 1 = A - 2B \\ 1 = 2A \end{array} \right\} \Rightarrow A = \frac{1}{2} \left. \begin{array}{l} \\ \\ \end{array} \right\} C = \frac{1}{2}$$

$$B = -\frac{1}{4}$$

$$\therefore \int_0^1 \frac{dx}{(x+1)(x^2+1)} = \frac{1}{2} \int_0^1 \frac{dx}{x+1} + \left(-\frac{1}{4}\right) \int_0^1 \frac{2x dx}{x^2+1} + \frac{1}{2} \int_0^1 \frac{dx}{x^2+1}$$

$$= \frac{1}{2} \log|x+1| - \frac{1}{4} \log|x^2+1| + \frac{1}{2} \arctan x \Big|_0^1$$

$$= \frac{1}{4} \log(x+1)^2 - \frac{1}{4} \log|x^2+1| + \frac{1}{2} \arctan x \Big|_0^1$$

$$= \frac{1}{4} \log \frac{(x+1)^2}{x^2+1} + \frac{1}{2} \arctan x \Big|_0^1$$

$$= \frac{1}{4} \log \frac{(1+1)^2}{1+1} + \frac{1}{2} \arctan 1 - \frac{1}{4} \log \frac{1}{1} + \frac{1}{2} \arctan 0$$

$$= \frac{1}{4} \log 2 + \frac{1}{2} \frac{\pi}{4} - \frac{1}{4} \cdot 0 - \frac{1}{2} \cdot 0$$

$$= \frac{\log 2}{4} + \frac{\pi}{8}$$

# A. HEAVISIDE METHOD

quick method to use, but  
only for linear factors

.22-0  
26-1  
N25-1

EX # ~~486~~ ~~487~~ ~~488~~ ~~489~~ ~~490~~ ~~491~~ ~~492~~ ~~493~~ ~~494~~ ~~495~~ ~~496~~ ~~497~~ ~~498~~ ~~499~~ ~~500~~ ~~501~~ ~~502~~ ~~503~~ ~~504~~ ~~505~~ ~~506~~ ~~507~~ ~~508~~ ~~509~~ ~~510~~ ~~511~~ ~~512~~ ~~513~~ ~~514~~ ~~515~~ ~~516~~ ~~517~~ ~~518~~ ~~519~~ ~~520~~ ~~521~~ ~~522~~ ~~523~~ ~~524~~ ~~525~~ ~~526~~ ~~527~~ ~~528~~ ~~529~~ ~~530~~ ~~531~~ ~~532~~ ~~533~~ ~~534~~ ~~535~~ ~~536~~ ~~537~~ ~~538~~ ~~539~~ ~~540~~ ~~541~~ ~~542~~ ~~543~~ ~~544~~ ~~545~~ ~~546~~ ~~547~~ ~~548~~ ~~549~~ ~~550~~ ~~551~~ ~~552~~ ~~553~~ ~~554~~ ~~555~~ ~~556~~ ~~557~~ ~~558~~ ~~559~~ ~~560~~ ~~561~~ ~~562~~ ~~563~~ ~~564~~ ~~565~~ ~~566~~ ~~567~~ ~~568~~ ~~569~~ ~~570~~ ~~571~~ ~~572~~ ~~573~~ ~~574~~ ~~575~~ ~~576~~ ~~577~~ ~~578~~ ~~579~~ ~~580~~ ~~581~~ ~~582~~ ~~583~~ ~~584~~ ~~585~~ ~~586~~ ~~587~~ ~~588~~ ~~589~~ ~~590~~ ~~591~~ ~~592~~ ~~593~~ ~~594~~ ~~595~~ ~~596~~ ~~597~~ ~~598~~ ~~599~~ ~~600~~ ~~601~~ ~~602~~ ~~603~~ ~~604~~ ~~605~~ ~~606~~ ~~607~~ ~~608~~ ~~609~~ ~~610~~ ~~611~~ ~~612~~ ~~613~~ ~~614~~ ~~615~~ ~~616~~ ~~617~~ ~~618~~ ~~619~~ ~~620~~ ~~621~~ ~~622~~ ~~623~~ ~~624~~ ~~625~~ ~~626~~ ~~627~~ ~~628~~ ~~629~~ ~~630~~ ~~631~~ ~~632~~ ~~633~~ ~~634~~ ~~635~~ ~~636~~ ~~637~~ ~~638~~ ~~639~~ ~~640~~ ~~641~~ ~~642~~ ~~643~~ ~~644~~ ~~645~~ ~~646~~ ~~647~~ ~~648~~ ~~649~~ ~~650~~ ~~651~~ ~~652~~ ~~653~~ ~~654~~ ~~655~~ ~~656~~ ~~657~~ ~~658~~ ~~659~~ ~~660~~ ~~661~~ ~~662~~ ~~663~~ ~~664~~ ~~665~~ ~~666~~ ~~667~~ ~~668~~ ~~669~~ ~~670~~ ~~671~~ ~~672~~ ~~673~~ ~~674~~ ~~675~~ ~~676~~ ~~677~~ ~~678~~ ~~679~~ ~~680~~ ~~681~~ ~~682~~ ~~683~~ ~~684~~ ~~685~~ ~~686~~ ~~687~~ ~~688~~ ~~689~~ ~~690~~ ~~691~~ ~~692~~ ~~693~~ ~~694~~ ~~695~~ ~~696~~ ~~697~~ ~~698~~ ~~699~~ ~~700~~ ~~701~~ ~~702~~ ~~703~~ ~~704~~ ~~705~~ ~~706~~ ~~707~~ ~~708~~ ~~709~~ ~~710~~ ~~711~~ ~~712~~ ~~713~~ ~~714~~ ~~715~~ ~~716~~ ~~717~~ ~~718~~ ~~719~~ ~~720~~ ~~721~~ ~~722~~ ~~723~~ ~~724~~ ~~725~~ ~~726~~ ~~727~~ ~~728~~ ~~729~~ ~~730~~ ~~731~~ ~~732~~ ~~733~~ ~~734~~ ~~735~~ ~~736~~ ~~737~~ ~~738~~ ~~739~~ ~~740~~ ~~741~~ ~~742~~ ~~743~~ ~~744~~ ~~745~~ ~~746~~ ~~747~~ ~~748~~ ~~749~~ ~~750~~ ~~751~~ ~~752~~ ~~753~~ ~~754~~ ~~755~~ ~~756~~ ~~757~~ ~~758~~ ~~759~~ ~~760~~ ~~761~~ ~~762~~ ~~763~~ ~~764~~ ~~765~~ ~~766~~ ~~767~~ ~~768~~ ~~769~~ ~~770~~ ~~771~~ ~~772~~ ~~773~~ ~~774~~ ~~775~~ ~~776~~ ~~777~~ ~~778~~ ~~779~~ ~~780~~ ~~781~~ ~~782~~ ~~783~~ ~~784~~ ~~785~~ ~~786~~ ~~787~~ ~~788~~ ~~789~~ ~~790~~ ~~791~~ ~~792~~ ~~793~~ ~~794~~ ~~795~~ ~~796~~ ~~797~~ ~~798~~ ~~799~~ ~~800~~ ~~801~~ ~~802~~ ~~803~~ ~~804~~ ~~805~~ ~~806~~ ~~807~~ ~~808~~ ~~809~~ ~~810~~ ~~811~~ ~~812~~ ~~813~~ ~~814~~ ~~815~~ ~~816~~ ~~817~~ ~~818~~ ~~819~~ ~~820~~ ~~821~~ ~~822~~ ~~823~~ ~~824~~ ~~825~~ ~~826~~ ~~827~~ ~~828~~ ~~829~~ ~~830~~ ~~831~~ ~~832~~ ~~833~~ ~~834~~ ~~835~~ ~~836~~ ~~837~~ ~~838~~ ~~839~~ ~~840~~ ~~841~~ ~~842~~ ~~843~~ ~~844~~ ~~845~~ ~~846~~ ~~847~~ ~~848~~ ~~849~~ ~~850~~ ~~851~~ ~~852~~ ~~853~~ ~~854~~ ~~855~~ ~~856~~ ~~857~~ ~~858~~ ~~859~~ ~~860~~ ~~861~~ ~~862~~ ~~863~~ ~~864~~ ~~865~~ ~~866~~ ~~867~~ ~~868~~ ~~869~~ ~~870~~ ~~871~~ ~~872~~ ~~873~~ ~~874~~ ~~875~~ ~~876~~ ~~877~~ ~~878~~ ~~879~~ ~~880~~ ~~881~~ ~~882~~ ~~883~~ ~~884~~ ~~885~~ ~~886~~ ~~887~~ ~~888~~ ~~889~~ ~~890~~ ~~891~~ ~~892~~ ~~893~~ ~~894~~ ~~895~~ ~~896~~ ~~897~~ ~~898~~ ~~899~~ ~~900~~ ~~901~~ ~~902~~ ~~903~~ ~~904~~ ~~905~~ ~~906~~ ~~907~~ ~~908~~ ~~909~~ ~~910~~ ~~911~~ ~~912~~ ~~913~~ ~~914~~ ~~915~~ ~~916~~ ~~917~~ ~~918~~ ~~919~~ ~~920~~ ~~921~~ ~~922~~ ~~923~~ ~~924~~ ~~925~~ ~~926~~ ~~927~~ ~~928~~ ~~929~~ ~~930~~ ~~931~~ ~~932~~ ~~933~~ ~~934~~ ~~935~~ ~~936~~ ~~937~~ ~~938~~ ~~939~~ ~~940~~ ~~941~~ ~~942~~ ~~943~~ ~~944~~ ~~945~~ ~~946~~ ~~947~~ ~~948~~ ~~949~~ ~~950~~ ~~951~~ ~~952~~ ~~953~~ ~~954~~ ~~955~~ ~~956~~ ~~957~~ ~~958~~ ~~959~~ ~~960~~ ~~961~~ ~~962~~ ~~963~~ ~~964~~ ~~965~~ ~~966~~ ~~967~~ ~~968~~ ~~969~~ ~~970~~ ~~971~~ ~~972~~ ~~973~~ ~~974~~ ~~975~~ ~~976~~ ~~977~~ ~~978~~ ~~979~~ ~~980~~ ~~981~~ ~~982~~ ~~983~~ ~~984~~ ~~985~~ ~~986~~ ~~987~~ ~~988~~ ~~989~~ ~~990~~ ~~991~~ ~~992~~ ~~993~~ ~~994~~ ~~995~~ ~~996~~ ~~997~~ ~~998~~ ~~999~~ ~~1000~~

$$\frac{x^2+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

mult by  $x-1$

$$\frac{x^2+1}{(x-2)(x-3)} = A + \frac{B(x-1)}{x-2} + \frac{C(x-1)}{x-3}$$

let  $x=1$

$$\frac{1+1}{(-1)(-2)} = A = \frac{2}{+2} = 1$$

mult by  $x-2$

$$\frac{x^2+1}{(x-1)(x-3)} = \frac{A(x-2)}{x-1} + B + \frac{C(x-2)}{x-3}$$

let  $x=2$

$$\frac{5}{(1)(-1)} = B = -5$$

mult by  $x-3$

$$\frac{x^2+1}{(x-1)(x-2)} = \frac{A(x-3)}{x-1} + \frac{B(x-3)}{x-2} + C$$

let  $x=3$

$$\frac{10}{2 \cdot 1} = C = 5$$

This is sometimes called the "cover-up" method, because, in practice, people just "cover-up" one of the factors in the denominator while evaluating the other factors.

eg.

$$\frac{x^2+1}{(x-1)(x-2)(x-3)}$$

Cover up  $x-1$  while evaluating the rest of the expression at  $x=1$ .

$$\frac{1+1}{(1-2)(1-3)} = \frac{2}{-1 \cdot -2} = \frac{2}{2} = 1 = \underline{A}$$

etc.

EXAMPLES

do this + skip part 1 next page?

~~EX 5 p 485~~  
 cf. EX 6 p 487  
 cf. EX 3 p 474  
 divide first.

$$\int \frac{x^5 - x^4 - 3x + 5}{x^4 - 2x^3 + 2x^2 - 2x + 1} dx$$

$$\begin{array}{r} x+1 \\ \hline x^4 - 2x^3 + 2x^2 - 2x + 1 \mid x^5 - x^4 + 0 + 0 - 3x + 5 \\ \underline{x^5 - 2x^4 + 2x^3 - 2x^2 + x} \\ x^4 - 2x^3 + 2x^2 - 4x + 5 \\ \underline{x^4 - 2x^3 + 2x^2 - 2x + 1} \\ -2x + 4 \end{array}$$

$$\therefore = x+1 + \frac{-2x+4}{x^4 - 2x^3 + 2x^2 - 2x + 1}$$

Can we factor the denom? Try dividing by  $x-1$

$$\begin{array}{r} x^3 - x^2 + x - 1 \\ x-1 \mid x^4 - 2x^3 + 2x^2 - 2x + 1 \\ \underline{x^4 - x^3} \\ -x^3 + 2x^2 - 2x + 1 \\ \underline{-x^3 + x^2} \\ x^2 - 2x + 1 \end{array} \quad \begin{array}{l} = (x-1)(x^2) + x-1 \\ = (x-1)(x^2+1) \end{array}$$

$$x^2 - 2x + 1 = (x-1)^2$$

cf. EX 4 p 474-5

$$\therefore x^4 - 2x^3 + 2x^2 - 2x + 1 = (x-1)^2(x^2+1)$$

skip?

$$\frac{-2x+4}{x^4 - 2x^3 + 2x^2 - 2x + 1} = \frac{Ax}{x^2+1} + \frac{B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$\begin{aligned} -2x+4 &= Ax(x-1)^2 + B(x-1)^2 + C(x-1)(x^2+1) + D(x^2+1) \\ &= Ax(x^2-2x+1) + B(x^2-2x+1) + C(x^3-x^2+x-1) + D(x^2+1) \\ &= Ax^3 - 2Ax^2 + Ax + Bx^2 - 2Bx + B + Cx^3 - Cx^2 + Cx - C + Dx^2 + D \\ &= x^3(A+C) + x^2(-2A+B-C+D) + x(A-2B+C) + (B-C+D) \end{aligned}$$

$$\begin{array}{l} 0 = A+C \\ 0 = -2A+B-C+D \\ -2 = A-2B+C \\ 4 = B-C+D \end{array} \quad \begin{array}{l} 4 = 2A \Rightarrow \underline{A=2} \\ \underline{C=-2} \\ -2 = 2 - 2B - 2 \\ \underline{B=1} \end{array} \quad \begin{array}{l} 4 = 1 + 2 + D \\ \underline{D=1} \end{array}$$

$$\therefore \int \frac{-2x+4}{x^4-2x^3+2x^2-2x+1} dx = 2 \int \frac{x dx}{x^2+1} + \int \frac{dx}{x^2+1} + (-2) \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

$I_1$                        $I_2$                        $I_3$                        $I_4$

$$I_1: 2 \int \frac{x dx}{x^2+1} \quad \begin{array}{l} u=x^2+1 \\ du=2x dx \end{array} = \int \frac{du}{u} = \log |u| + C_1 = \log |x^2+1| + C_1$$

$$I_2: \int \frac{dx}{x^2+1} = \arctan x + C_2$$

$$I_3: -2 \int \frac{dx}{x-1} \quad \begin{array}{l} u=x-1 \\ du=dx \end{array} = -2 \int \frac{du}{u} = -2 \log |u| + C_3 = -2 \log |x-1| + C_3$$

$$I_4: \int \frac{dx}{(x-1)^2} \quad \begin{array}{l} u=x-1 \\ du=dx \end{array} = \int \frac{du}{u^2} = \int u^{-2} du = \frac{u^{-1}}{-1} + C_4 = -\frac{1}{x-1} + C_4$$

$$\therefore I_1 + I_2 + I_3 + I_4 = \log |x^2+1| + \arctan x - 2 \log |x-1| - \frac{1}{x-1} + C$$

$$= \log \frac{x^2+1}{(x-1)^2} + \arctan x - \frac{1}{x-1} + C_5$$

$$\therefore \int \frac{x^5-x^4-3x+5}{x^4-2x^3+2x^2-2x+1} dx = \int x+1 dx + \int \frac{-2x+4}{x^4-2x^3+2x^2-2x+1} dx$$

$$= \frac{x^2}{2} + x + \log \frac{x^2+1}{(x-1)^2} + \arctan x + \frac{1}{1-x} + C //$$

WHY ALL TERMS.

$$\frac{A}{x^2+1} + \frac{Bx}{x^2+1} + \frac{C}{(x^2+1)^2} + \frac{Dx}{(x^2+1)^2} = \frac{x^3}{(x^2+1)^2}$$

---

$$\frac{1}{x^2+1} + \frac{x}{(x^2+1)^2} = \frac{x^2+1}{(x^2+1)^2} + \frac{x}{(x^2+1)^2} = \frac{x^2+x+1}{(x^2+1)^2}$$

---

$$\frac{x}{x^2+1} + \frac{1}{(x^2+1)^2} = \frac{x(x^2+1)}{(x^2+1)^2} + \frac{1}{(x^2+1)^2} = \frac{x^3+x+1}{(x^2+1)^2}$$

---

$$\frac{1}{x^2+1} + \frac{x}{x^2+1} + \frac{1}{(x^2+1)^2} + \frac{x}{(x^2+1)^2} = \frac{(x^2+1) + (x^3+x) + 1 + x}{(x^2+1)^2}$$
$$= \frac{x^3 + x^2 + 2x + 2}{(x^2+1)^2}$$

---

$$\frac{1}{x^2+1} + \frac{x}{x^2+1} - \frac{1}{(x^2+1)^2} - \frac{x}{(x^2+1)^2} = \frac{(x^2+1) + (x^3+x) - 1 - x}{(x^2+1)^2}$$
$$= \frac{x^3 + x^2}{(x^2+1)^2}$$

~~P3 487~~  
~~422~~

~~NST~~  
~~8A~~ p 479

#13

$$\int_4^8 \frac{x dx}{x^2 - 2x - 3}$$

$$x^2 - 2x - 3 = (x-3)(x+1)$$

$$\frac{x}{x^2 - 2x - 3} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$x = Ax + A + Bx - 3B$$

x:  $1 = A + B$

const:  $0 = A - 3B$

$$1 = 4B \Rightarrow B = \frac{1}{4}$$

$$\Rightarrow A = \frac{3}{4}$$

$$\begin{aligned} \int_4^8 \frac{x dx}{x^2 - 2x - 3} &= \frac{3}{4} \int_4^8 \frac{dx}{x-3} + \frac{1}{4} \int_4^8 \frac{dx}{x+1} \\ &= \frac{3}{4} \log|x-3| + \frac{1}{4} \log|x+1| \Big|_4^8 \end{aligned}$$

$$= \frac{3}{4} \log|5| + \frac{1}{4} \log 9 - \frac{3}{4} \log 1 - \frac{1}{4} \log 5$$

$$= \frac{1}{2} \log 5 + \frac{1}{2} \cdot \frac{1}{2} \log 3^2$$

$$= \frac{1}{2} \log 5 + \frac{1}{2} \log 3$$

$$= \frac{1}{2} \log 15 = \log \sqrt{15}$$

ex

$$\int \sec \theta d\theta = \int \frac{d\theta}{\cos \theta} = \int \frac{\cos \theta}{\cos^2 \theta} d\theta = \int \frac{\cos \theta d\theta}{1 - \sin^2 \theta}$$

Let  $x = \sin \theta$   
 $dx = \cos \theta d\theta$

$$= \int \frac{dx}{1-x^2} \quad (\text{of ps 450 \# 11}) \quad \text{? L.R.}$$

$$1-x^2 = (1-x)(1+x)$$

$$\frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$1 = A(1+x) + B(1-x) = A + Ax + B - Bx$$

$$= (A-B)x + (A+B)$$

$\therefore 0 = A - B$       coef of  $x$

$1 = A + B$       const.

addg  $1 = 2A \Rightarrow A = \frac{1}{2}$

$B = \frac{1}{2}$

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \int \frac{dx}{1-x} + \frac{1}{2} \int \frac{dx}{1+x}$$

$u = 1-x \quad v = 1+x$   
 $du = -dx \quad dv = dx$

$$= -\frac{1}{2} \int \frac{du}{u} + \frac{1}{2} \int \frac{dv}{v}$$

$$= -\frac{1}{2} \log |u| + \frac{1}{2} \log |v| + C$$

$$= -\frac{1}{2} \log |1-x| + \frac{1}{2} \log |1+x| + C$$

$$= \frac{1}{2} \log \left| \frac{1+x}{1-x} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{1+\sin \theta}{1-\sin \theta} \right| + C$$

$$= \log \sqrt{\frac{1+\sin \theta}{1-\sin \theta}} + C$$

$$= \log \sqrt{\frac{(1+\sin \theta)^2}{1-\sin^2 \theta}} + C$$

$$= \log \sqrt{\frac{(1+\sin \theta)^2}{\cos^2 \theta}} + C$$

$$= \log \left( \frac{1+\sin \theta}{\cos \theta} \right) + C$$

$$= \log |\sec \theta + \tan \theta| + C //$$

mult by  $\frac{1+\sin \theta}{1+\sin \theta}$

483 compl. sq

H 1, 9, 11

HW 1, 3, 5, 9

490 # 23, 25, 29

506 # 2, 9