INTEGRATION BY PARTS CONT'D (8.1)

$$\int u\,dv = uv - \int v\,du$$

Choose u, dv so that
the new integral is "simpler"

NOTE: $\int \frac{dv}{u} \neq \int u\,dv$

$\therefore$ cannot use parts

$$\int \frac{dx}{x^2+1} \rightarrow u = x^2+1 \quad dv = dx$$

PARTS USING TABLES

We can simplify our computation of answers if we notice that repeated use of the "parts rule" will reduce one factor to zero. I.e. if we are given $\int f(x)\,g(x)\,dx$ and notice that taking derivatives of $f(x)$ will eventually give us zero, we can set things up in a Table as follows.

EG. $\int x^2 e^x\,dx$ (seen earlier in "repeating the rule")

7th ~~EX 6 p 454~~
8th ~~EX~~ EX 7 p ~~456~~ 466

$= \int f(x)\,g(x)\,dx$

let $x^2 = f(x)$ and $g(x) = e^x$

i.e. $u = x^2$ and $e^x dx = dv$

We notice and choose $f(x) = u$ s.t. repeated differentiation will give us zero!

(1st) fill in table until get zero under $f(x)$

$f(x)$ (take derivatives)	sign	$g(x)$ (take integrals)
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

(2nd) draw diagonals from f column to g column with alternating signs

(3rd) answer is the product of corresponding columns with sign (plus C).

$$\int x^2 e^x\,dx = +x^2e^x - 2xe^x + 2e^x + C$$

~~7th EX 7 p 455~~

~~8th EX 8 p 491~~

~~EX 8 p 459~~

$\int x^3 \sin x \, dx$

$u = f(x)$		$g(x) = \sin x$ / dv
x^3	$+$	$\sin x$
$3x^2$	$-$	$-\cos x$
$6x$	$+$	$-\sin x$
6	$-$	$\cos x$
0		$\sin x$

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C \;//$$

EXAMPLES

cf ~~1, 455~~ #3 of ~~459~~ 467

here $\theta = ax$

$$\int x^2 \cos ax \, dx$$

we can use tabular method

$f(x) = u$		$g(x) = dv$
x^2		$\cos ax$
$2x$	$+$	$\frac{\sin ax}{a}$
2	$-$	$-\frac{\cos ax}{a^2}$
0	$+$	$-\frac{\sin ax}{a^3}$

$$= \frac{x^2 \sin ax}{a} + \frac{2x \cos ax}{a^2} - \frac{2 \sin ax}{a^3} + C$$

—or— NON-Tabular

$u = x^2 \qquad dv = \cos ax \, dx$

$du = 2x \, dx \qquad v = \frac{\sin ax}{a}$

$$= \frac{x^2 \sin ax}{a} - \left[\frac{2}{a}\int x \sin ax \, du\right]$$

$u = x \qquad dv = \sin ax \, dx$

$du = dx \qquad v = \frac{-\cos ax}{a}$

$$= \frac{x^2 \sin ax}{a} - \frac{2}{a}\left[\frac{-x \cos ax}{a} + \frac{1}{a}\int \cos ax \, dx\right]$$

$$= \frac{x^2 \sin ax}{a} + \frac{2x \cos ax}{a^2} - \frac{2}{a^2}\int \cos ax \, dx$$

$$= \frac{x^2 \sin ax}{a} + \frac{2x \cos ax}{a^2} - \frac{2 \sin ax}{a^3} + C$$

MONKEY WRENCHES (PARTS 8.2)

skip?

$$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \cos x \csc x\,dx$$

$u = \csc x \qquad dv = \cos x\,dx$

$du = -\csc x \cot x\,dx \qquad v = \sin x$

$$= \sin x \csc x + \int \sin x \csc x \cot x\,dx$$

But $\csc x = \frac{1}{\sin x}$

$$= 1 + \int \cot x\,dx$$

subtracting $\int \cot x\,dx$ from both sides.

$\underline{0 = 1}$?

Try simple substitution.

$$\int \cot x\,dx = \int \frac{\cos x\,dx}{\sin x}$$

$u = \sin x$

$du = + \cos x\,dx$

$+du = \cos x\,dx$

$$= + \int \frac{du}{u}$$

$$= + \log|u| + c = + \log|\sin x| + C$$

Parts will "work" if we include the <u>integration constants.</u>

e.g. $v = \sin x + c,$

EXAMPLE with constant

05 Sep

② $\int \log(x+1)\,dx$ $\quad u = \log(x+1) \quad dv = dx$

$du = \dfrac{dx}{x+1} \quad v = x + C_1$

$$= [\log(x+1)][x+C_1] - \int \frac{x+C_1}{x+1}\,dx$$

life would be simpler if $C_1 = 1$, Since it drops out, let's assign it as 1.

$$= [\log(x+1)][x+1] - \int \frac{x+1}{x+1}\,dx$$

$$= (x+1)\log(x+1) - \int dx$$

$$= (x+1)\log(x+1) - x + C_2$$

Kp 29?

"REDUCTION" FORMULAE

~~EX7~~ is ~~what~~ ~~457~~ 464

EX5

$$\int \cos^n x\,dx = \int \cos^{n-1}x \cos x\,dx \qquad u = \cos^{n-1}x \qquad dv = \cos x\,dx$$

$$du = (n-1)(\cos^{n-2}x)(-\sin x)\,dx \qquad v = \sin x$$

$$= \sin x \cos^{n-1}x + (n-1)\int \sin^2 x \cos^{n-2}x\,dx$$

$$= \sin x \cos^{n-1}x + (n-1)\int (1-\cos^2 x)\cos^{n-2}x\,dx$$

$$= \sin x \cos^{n-1}x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\underbrace{\int \cos^n x\,dx}_{\text{what we started with! take it to other side.}}$$

$$n\int \cos^n x\,dx = \sin x \cos^{n-1}x + (n-1)\int \cos^{n-2}x\,dx$$

dividing by n

$$\int \cos^n x\,dx = \frac{\cos^{n-1}x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

This allows us to re-write $\int \cos^n x\,dx$ in terms of an integral with a "reduced" power, we can repeat its use until the power of the last integral is 0 or 1.