REVIEW

$$\frac{d}{dx}(x) = 1 \Rightarrow \int dx = \int 1\,dx = x + c$$

$$\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x\,dx = \sin x + c$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2} \Rightarrow \int \frac{dx}{1+x^2} = \arctan x + c$$

~~pg 389 #19 (6.3)~~

$$y = x(\sin^{-1} x)^2 - 2x + 2\sqrt{1-x^2}\ \arcsin x$$

$$\frac{dy}{dx} = (\sin^{-1} x)^2$$

$$\Rightarrow \int (\sin^{-1}x)^2\,dx = x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\ \arcsin x + C$$

INTEGRATION BY PARTS (8.1)

Start with the product rule for differentiation.

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

or $d(uv) = u\,dv + v\,du$

or $u\,dv = d(uv) - v\,du$

Integrating, we get

$$\int u\,dv = \int d(uv) - \int v\,du$$

But $\int d(uv) = uv + C$

$$\therefore \quad \int u\,dv = uv - \int v\,du$$

Rule for int by parts.

This doesn't look like much. But often the second integral is <u>much easier</u> to work with than the first.

If not something is wrong.

1. perhaps you had the wrong choice for u & v
2. perhaps this method won't work.

Main challenge: How to choose <u>u</u> and <u>dv</u> so that things <u>don't get worse</u>!

EXAMPLES

~~7F EX 3 p 453~~

① ~~8 EX 4 p 488~~ EX 2 p 455

$$\int \log x \, dx \qquad \text{let } u = \log x \qquad dv = dx$$

$$\text{then } du = \frac{dx}{x} \qquad v = x + C_1$$

$$\therefore \int \log x \, dx = \int u \, dv$$

$$= uv - \int v \, du = (\log x)(x + C_1) - \int (x + C_1) \frac{dx}{x}$$

$$= x \log x + C_1 \log x - \int dx - C_1 \int \frac{dx}{x}$$

$$= x \log x + C_1 \cancel{\log} x - x - C_1 \cancel{\log} x + C_2$$

$$= \underline{x \log x - x + C_2}$$

NOTE that C_1 disappeared. This always happens.

$\therefore$ we usually drop it for calculations.

But once in a while, it helps simplify the notation.

05 skip

09 skip

② $\int \log(x+1) \, dx \qquad u = \log(x+1) \qquad dv = dx$

$$du = \frac{dx}{x+1} \qquad v = x + C_1$$

$$= [\log(x+1)][x + C_1] - \int \frac{x + C_1}{x+1} dx$$

Life would be simpler if $C_1 = 1$. Since it drops out, let's assign it as 1.

$$= [\log(x+1)][x+1] - \int \frac{x+1}{x+1} dx$$

$$= (x+1) \log(x+1) - \int dx$$

$$= \underline{(x+1) \log(x+1) - x + C_2}$$

INVERSE TRIG FUNCTIONS

~~7th p. 485 #7~~
~~8th 491 #7~~
~~459~~ #11
467

$$\int \arctan x \, dx$$

let $u = \arctan x$ $\quad dv = dx$

$du = \dfrac{dx}{1+x^2}$ $\quad v = x$

$$= x \arctan x - \int \frac{x\,dx}{1+x^2}$$

$w = 1+x^2$

$dw = 2x\,dx$

$\dfrac{dw}{2} = x\,dx$

$$= x \arctan x - \frac{1}{2} \int \frac{dw}{w}$$

$$= x \arctan x - \frac{1}{2} \log |w| + C$$

$$= x \arctan x - \frac{1}{2} \log(1+x^2) + C \quad //$$

IMPORTANT COMMENT

To reiterate the comment given after the rule for parts, the new integral should be simpler than the old. If not, perhaps you choose the wrong things as u and dv.

EX. ~~8th p. 488~~ ~~7th p. 452~~ $\quad \int x e^x \, dx$

<u>WRONG</u>

$u = e^x$ $\quad dv = x\,dx$

$du = e^x dx$ $\quad v = \dfrac{x^2}{2}$

$$= \frac{e^x x^2}{2} - \frac{1}{2} \underbrace{\int x^2 e^x \, dx}$$

since this has a higher power of x than the original, this is worse than where we started.

<u>RIGHT</u>

$u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = e^x$

$$= x e^x - \int e^x \, dx$$

$$= x e^x - e^x + C \quad //$$

PRODUCTIVELY "GETTING NOWHERE"

~~#1~~ PS ~~439~~ #5
467

$$\int_1^2 x \log x \, dx$$

let $u = x\log x$ $\quad dv = dx$

$du = \left(\frac{x}{x} + 1\cdot \log x\right)dx \quad v = x$

$= (1 + \log x)\, dx$

$$= x \cdot x\log x - \int (x + x\log x)\, dx$$

$$= x^2 \log x - \int x\,dx - \underbrace{\int x \log x \, dx}_{\text{original integral}}$$

$$= x^2\log x - \frac{x^2}{2} + C_1 - \underbrace{\int x\log x\, dx}_{\text{take to other side as in } \cos^n x \text{ reduction formula}}$$

$$\Rightarrow 2\int x\log x\,dx = x^2 \log x - \frac{x^2}{2} + C_1$$

$$\Rightarrow \int x \log x\,dx = \left.\frac{x^2\log x}{2} - \frac{x^2}{4} + C_2\right]_1^2$$

NOTE: "WHAT IF" we took a different u and v.
i.e.

$$\int x \log x\, dx \qquad u = \log x \qquad dv = x\,dx$$

$$du = \frac{dx}{x} \qquad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2}\log x - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{dx}{\cancel{x}}$$

$$= \frac{x^2\log x}{2} - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2 \log x}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

$$= \frac{x^2\log x}{2} - \frac{x^2}{4} + C$$

SAME AS ABOVE !!!

REPEATING THE RULE

"If at first you don't succeed, try, try, again."

~~EX 5 p 489~~

EX 3 p ~~456~~ 463

$$\int x^2 e^x \, dx \qquad u = x^2 \quad dv = e^x \, dx \qquad du = 2x \, dx \quad v = e^x$$

$$= x^2 e^x - 2\int x e^x \, dx$$

this integral is <u>better</u> than original – lower power of x! but still we can't integrate it

$$u = x \quad dv = e^x \, dx \qquad du = dx \quad v = e^x$$

$$= x^2 e^x - 2\left[x e^x - \int e^x \, dx\right]$$

$$= x^2 e^x - 2\left[x e^x - e^x + C_1\right]$$

$$= x^2 e^x - 2x e^x + 2e^x + C \;//$$

$$u = (\arcsin x)^2 \qquad dv = dx$$
$$du = \frac{2 \arcsin x}{\sqrt{1-x^2}}\, dx \qquad v = x$$

$$x(\arcsin x)^2 - 2\int \frac{x \arcsin x}{\sqrt{1-x^2}}\, dx$$

$$u = \arcsin x \qquad dv = \frac{x}{\sqrt{1-x^2}}\, dx$$

$$du = \frac{1}{\sqrt{1-x^2}}\, dx$$

$$w = 1-x^2$$
$$dw = -2x\,dx$$
$$\frac{dw}{-2} = x\,dx$$

$$v = -\frac{1}{2}\int w^{-1/2}\, dw$$
$$= -\frac{1}{2}\frac{w^{1/2}}{1/2}$$
$$= -\sqrt{1-x^2}$$

$$= x(\arcsin x)^2 - 2\left[-\sqrt{1-x^2}\arcsin x + \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\, dx\right]$$

$$= x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$$