E. METHOD OF "WASHERS" ~~7th (5.4 - p 329) pg 331 (7th)~~ 6.1 p 369 0

~~6~~ 4

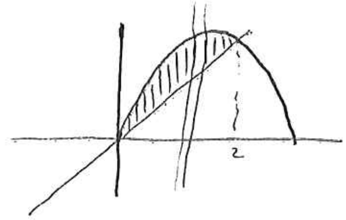
9-1

In these cases the "slice" is more like a washer than a circle.

EG. pg ~~245~~ 7th 328 # 3 (NOT in ~~8th Ed~~ 12)

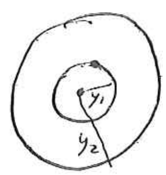
Rotate area between $y = 3x - x^2$ and $y = x$ around x-axis.

① when do they intersect?



$$x = 3x - x^2$$
$$0 = 2x - x^2$$
$$x(2 - x) = 0$$
$$\therefore x = 0 \text{ or } 2$$

② area of cross-section = washer



$y_1 = x$
$y_2 = 3x - x^2$

$y_2 = r_o$ (outer radius)
$y_1 = r_i$ (inner radius)

$$\text{Area of washer} = \pi y_2^2 - \pi y_1^2 = \pi(y_2^2 - y_1^2)$$

③ plug in general formula.

$$V = \int_0^2 \pi\big(\overbrace{(3x - x^2)^2}^{r_o^2} - \overbrace{x^2}^{r_i^2}\big)\,dx$$

$$= \pi \int_0^2 9x^2 - 6x^3 + x^4 - x^2 \, dx = \pi \int_0^2 8x^2 - 6x^3 + x^4 \, dx$$

$$= \pi \left[\frac{8x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5}\right]_0^2$$

$$= \pi \left[\frac{8 \cdot 8}{3} - \frac{6 \cdot 16}{4} + \frac{32}{5} - (0 - 0 + 0)\right]$$

$$= \pi \left[\frac{64}{3} - 24 + \frac{32}{5}\right] = \pi \left[\frac{\cancel{960}\,320 - 360 + 96}{15}\right]$$

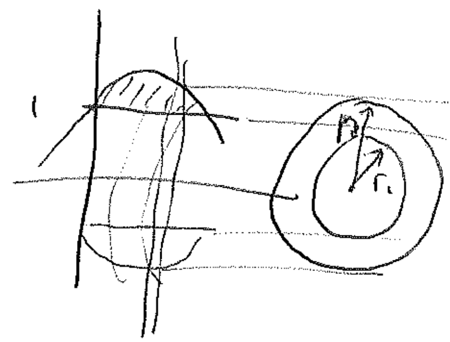
$$= \frac{56\pi}{15}$$

HW ~~pg 239 #1, 10, #14~~
~~pg 245 #5~~

EXAMPLE

Rotate area between $y = -x^2 + x + 1$ and $y = 1$ around x-axis and find volume

$$V = \int_{\ell_1}^{\ell_2} \underbrace{A(x)}_{\text{area of cross-sect}} \underbrace{dx}_{\text{width}}$$



$r_o = -x^2 + x + 1$

$r_i = 1$

$$= \pi \int_0^1 \overbrace{(-x^2+x+1)^2}^{r_o^2} - \overbrace{1^2}^{r_i^2} \, dx$$

Cross sect is a washer

limits?

$$-x^2 + x + 1 = 1$$
$$-x^2 + x = 0$$
$$x(1-x) = 0$$
$$\Rightarrow x = 0, 1$$

$$(-x^2+x+1)(-x^2+x+1)$$
$$= x^4 - x^3 - x^2$$
$$-x^3 + x^2 + x$$
$$-x^2 + x + 1$$
$$= x^4 - 2x^3 - x^2 + 2x + 1$$

$$= \pi \int_0^1 x^4 - 2x^3 - x^2 + 2x + \cancel{1} - \cancel{1} \, dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{2x^4}{4_{2}} - \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} \right]_0^1$$

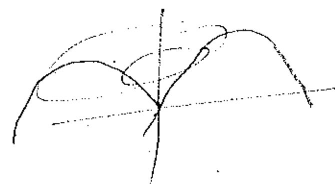
$$= \pi \left[\frac{1}{5} - \frac{1}{2} - \frac{1}{3} + 1 - (0) \right]$$

$$= \pi \left[\frac{6 - 15 - 10 + 30}{30} \right]$$

$$= \pi \left[\frac{6+5}{30} \right] = \underline{\underline{\frac{11\pi}{30}}}$$

EXAMPLE

$y = 3x - x^2$



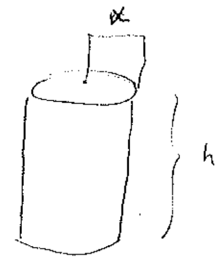
problem if use washers

6.2 p374 ~~7-1~~ 19-2

8
37
39

Fig 5.25 p 373

A. VOLUME BY CYLINDRICAL SHELLS



How do you calculate the volume of a cylinder? –
Slit it + unroll it.

If radius $= x$, circumference $= 2\pi x$

If height $= h$, surface area $= 2\pi x h$

If width $= \Delta x$, volume $= 2\pi x h \Delta x$

If height is function of radius, then volume is $2\pi x f(x)\,\Delta x$.

Now if we cut a solid into cylinders with the axis at the origin, then we can find the volume by adding up the volumes of the cylinders.

i.e. $$V \approx \sum_a^b 2\pi x f(x)\,\Delta x$$

or using same theory as before,

$$V = \lim_{\Delta x \to 0} \sum_a^b 2\pi x f(x)\,\Delta x = \int_a^b 2\pi x f(x)\,dx$$

B. EXAMPLE [~~sk~~ p 326, EX 6]

~~↑ p 2~~ ~~243-4~~ 341 - EX ~~4~~ 4 - "METHOD 3" (CYLIND. SHELLS)

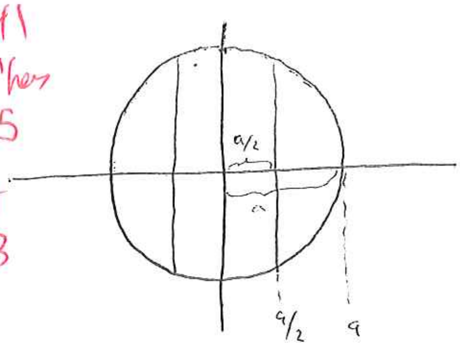
p 324 EX 6 of 6.2

p 381 # 41 "bead" has rad = 5 drill bit rad = 3

a circle ~~of~~ $x^2+y^2=a^2$ is rotated. $\boxed{a=2}$

a hole of diameter a is bored through the center.

What is the remaining volume?



$$y^2 = a^2 - x^2$$

$$y = \sqrt{a^2-x^2}$$

$$\text{height} = f(x) = 2\sqrt{a^2-x^2}$$

$$\therefore \text{Volume} = \int_{a/2}^{a} 2\pi x\, f(x)\, dx$$

$$= 2\pi \int_{a/2}^{a} x\, 2\sqrt{a^2-x^2}\, dx = 4\pi \int_{a/2}^{a} x\sqrt{a^2-x^2}\, dx$$

$$\left[\begin{array}{l} \text{let } a^2 - x^2 = u \\ -2x\,dx = du \quad \Rightarrow \quad x\,dx = -\frac{du}{2} \end{array}\right.$$

$$= \frac{4\pi}{-2}\int_{a/2}^{a} u^{\frac{1}{2}}\, du = -2\pi\left[\frac{2u^{3/2}}{3}\right]_{x=a/2}^{x=a}$$

$$= -\frac{4\pi}{3}(a^2-x^2)^{3/2}\Bigg]_{x=a/2}^{x=a}$$

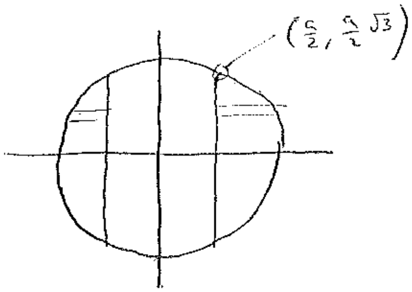
$$= -\frac{4\pi}{3}\left[0 - \left(a^2 - \frac{a^2}{4}\right)^{3/2}\right] = \frac{4\pi}{3}\left[\left(\frac{3a^2}{4}\right)^{3/2}\right]$$

$$= \frac{4\pi}{\cancel{3}}\,\frac{(\cancel{3}\sqrt{3})(a^3)}{\cancel{8}\,2} = \frac{\sqrt{3}}{2}\pi a^3$$

SAME PROBLEM - DIFF METHOD

- BY WASHERS - Pg ~~343~~ 343 - EX 2 - "METHOD 2"

7π / 326 EX 6



cut ⊥ to y-axis

$x^2 + y^2 = a^2$

if $x = a/2$

$\frac{a^2}{4} + y^2 = a^2$

$y^2 = \frac{3a^2}{4}$

$y = \frac{a\sqrt{3}}{2}$

Instead of integrating for $y = -\frac{a}{2}\sqrt{3}$ to $y = \frac{a}{2}\sqrt{3}$

Lets integrate from 0 to $y = \frac{a}{2}\sqrt{3}$ and double it.

$$Vol = 2\int_0^{\frac{a}{2}\sqrt{3}} \pi\left(x_2^2 - x_1^2\right)dy$$

$$= 2\pi\int_0^{(a/2)\sqrt{3}} x^2 - \left(\frac{a}{2}\right)^2 dy \qquad x^2 = a^2 - y^2$$

$$= 2\pi\int_0^{\frac{a\sqrt{3}}{2}} a^2 - y^2 - \frac{a^2}{4}\, dy$$

$$= 2\pi\int_0^{\frac{a\sqrt{3}}{2}} \frac{3a^2}{4} - y^2\, dy$$

$$= 2\pi\left[\frac{3a^2}{4}y - \frac{y^3}{3}\right]_0^{\frac{a\sqrt{3}}{2}}$$

$$= \cancel{2}\pi\left[\frac{3a^2}{4}\cdot\frac{a\sqrt{3}}{\cancel{2}} - \frac{1}{\cancel{3}}\frac{a^3\,\cancel{3}\sqrt{3}}{8\,4}\right]$$

$$= \pi\left[\frac{\cancel{8}a^3\sqrt{3}}{\cancel{4}\,2}\right] = \frac{\sqrt{3}}{2}\pi a^3$$