+ loaf of bread

6-1
8-3

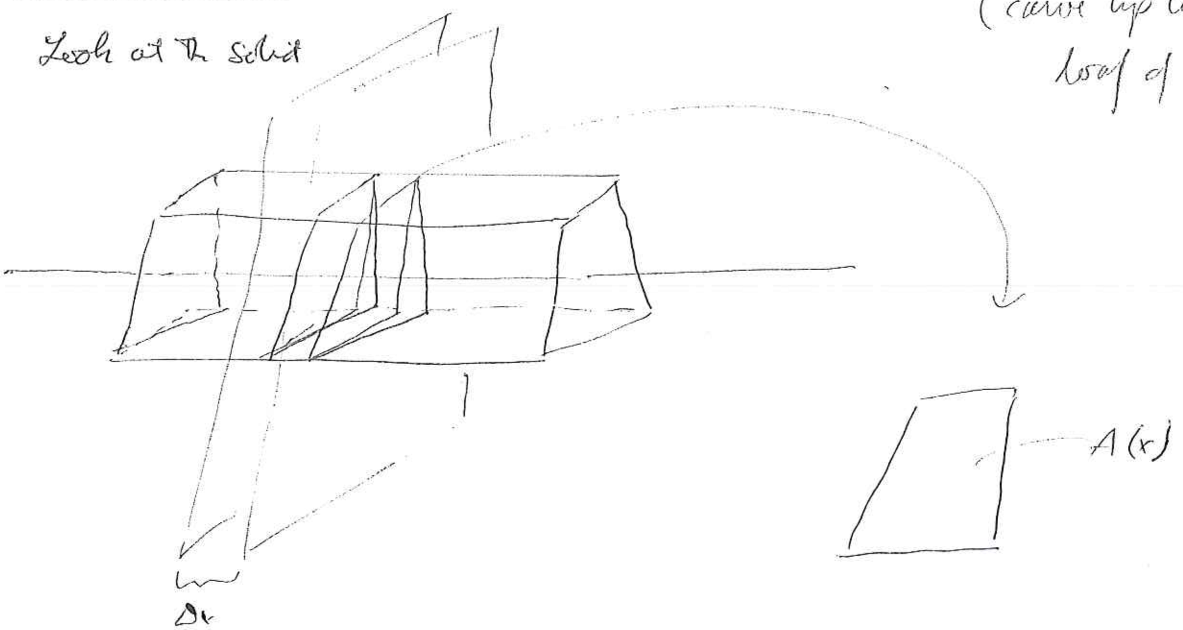
A VOLUMES AND INTEGRALS (6.1)

We can use a method similar to that used to approximate area under a curve in order to find the volume of a solid.

B. METHOD OF SLICES - GEN INTRO

Look at the solid

(carve up like loaf of bread)



$$\text{Volume of solid} \approx \sum_a^b A(x_i)\Delta x$$

using same theory as we did for formula for area under curves, we get.

$$V = \lim \sum_a^b A(x)\Delta x = \int_a^b A(x)\,dx.$$

6-1B
8-4
2

E.G. 337 # 49   p 319 # 35   p371 # 10

Base of solid is circle $x^2+y^2=a^2$
Cutting it by planes $\perp$ to y-axis gives isosceles right triangles
with one leg in base = circle.

Area $= \frac{b^2}{2}$

what is b?

$b = 2x$ and $x = \sqrt{a^2-y^2}$

$b = 2\sqrt{a^2-y^2}$ $b^2 = 4(a^2-y^2)$

$$\text{Vol} = \int_{-a}^{a} A(y)\,dy = \int_{-a}^{a} \frac{b^2}{2}\,dy = 2\int_{-a}^{a} a^2-y^2\,dy$$

$$= 2\left[a^2y - \frac{y^3}{3}\right]_{-a}^{a}$$

$$= 2\left[a^3 - \frac{a^3}{3} - \left(-a^3+\frac{a^3}{3}\right)\right]$$

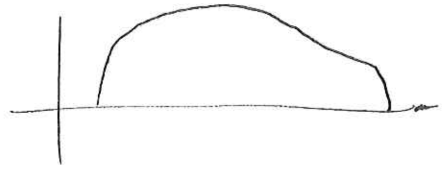
$$= 2\left[2a^3 - \frac{2a^3}{3}\right] = 4\left[\frac{2a^3}{3}\right] = \frac{8a^3}{3}$$

Styrofoam model

C VOLUME OF SOLID OF REVOLUTION

what happens when we revolve a curve around an axis → get solid.

EG.



→ get apple wire + dowel

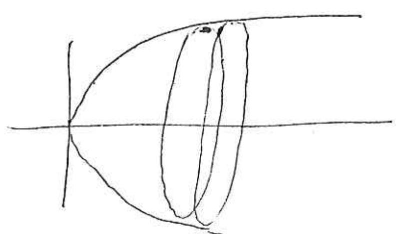
→ Take slice – look at area of cross section – disk.

→ circular → formula involves radius (y-height) and π.

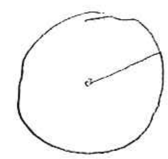
volume is Area times width.

~~Ex~~ 4
EX ~~5~~ 4 pg ~~328~~ 316 ~~pg 317 EX 3~~

take $y = \sqrt{x}$ rotated around x-axis from $x=0$ to $x=4$



cross section



radius $r = y = \sqrt{x}$

area circle $= \pi r^2$

$\therefore$ area slice $= \pi y^2 = \pi x = A(x)$

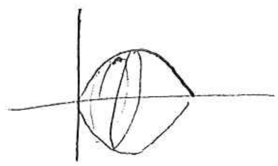
$$\therefore V = \int_0^4 A(x)\,dx = \int_0^4 \pi x\,dx$$

$$= \frac{\pi x^2}{2}\Big]_0^4 = \frac{16\pi}{2} = \underline{\underline{8\pi}}.$$

~~6~~ 8-6
4

E.G. p. ~~378~~ ~~239~~ p318 #2

area between $y = \sin x$ and $y = 0$ for $0 \le x \le \pi$ is rotated around x-axis.



$$r = y = \sin x$$

$$A(x) = \pi r^2 = \pi \sin^2 x$$

$$V = \int_0^\pi \pi \sin^2 x \, dx$$

$$= \frac{\pi}{2} \int_0^\pi 1 - \cos 2x \, dx = \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^\pi$$

$$= \frac{\pi}{2} \left[\left(\pi - \frac{0}{2} \right) - \left(0 - \frac{0}{2} \right) \right] = \underline{\underline{\frac{\pi^2}{2}}}$$

D. GENERAL PROCEDURE

1. Look at cross section – find area $\quad A(x)$
2. Multiply by width ($= \Delta x$) getting volume of the slice $\quad A(x)\Delta x$
3. Add up small volumes, getting approximate volume of solid. $\quad \sum A(x)\Delta x$
4. Change to an integral. $\quad \int_a^b A(x)\,dx.$