# Memorize #1

## Formulas

**Area Between 2 Curves** $f$ and $g$ on [a, b] where $f \geq g$.

$A = \int_a^b [f(x) - g(x)]dx$ **or** $A = \int_a^b [f(y) - g(y)]dy$ (for functions of $y$ only!)

(top curve – bottom curve) (right curve – left curve)

**Volumes of Revolution**

**Discs** $V = \pi \int_a^b r^2\, dx \ (or \ dy)$ **Washers** $V = \pi \int_a^b (R^2 - r^2) dx \ (or \ dy)$

**Volumes of Known Cross Sections**

$V = \int_a^b A\, dx \ or \ dy$ where $A$ represents the area of a representative cross section.

**Position – Velocity – Acceleration**

$s(t)$ or $x(t)$ – $v(t)$ – $a(t)$

$\frac{d}{dt} s(t) = v(t)$ $\int a(t)dt = v(t) + C$

$\frac{d}{dt} v(t) = a(t)$ $\int v(t)dt = s(t) + C$

$v(t) = 0$ implies particle at rest.
$v(t) > 0$ implies particle moving to right.
$v(t) < 0$ implies particle moving to left.

Speed = $|v(t)|$

Total Distance Traveled (from $t = a$ to $t = b$) = $\int_a^b |v(t)| dt$ *

* If you cannot use a calculator you must split this integral up where a change of direction occurs. (Make a velocity number line.)

**Tangent – Normal Lines**

Tangent: $y - y_1 = m(x - x_1)$ use the derivative to find m

Normal: $y - y_1 = \frac{-1}{m}(x - x_1)$

**Curve Sketching** for $y = f(x)$:

Max. or Min. Points: Relative: $f'(x) = 0$ or $f'(x)$ undefined. **Make an $f'$ number line.**

Absolute: At relative extrema **or endpoints** of a closed interval.

$f$ increasing: $f'(x) > 0$ $f$ decreasing: $f'(x) < 0$

Points of Inflection: $f''(x) = 0$ or $f''(x)$ undefined **Make an $f''$ number line.**

$f$ concave upward: $f''(x) > 0$ $f$ concave downward: $f''(x) < 0$

**CONTINUED**

**Average Value of a Function**: $f_{avg} = \frac{\int_a^b f(x)\,dx}{b-a}$ (average height on $[a, b]$)

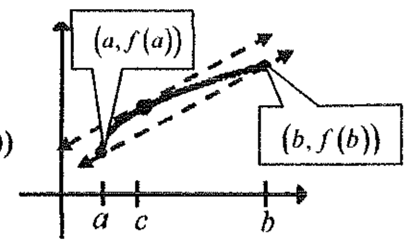
**Average Slope of a Function**: $m = \frac{f(b)-f(a)}{b-a}$ (this is just slope between two points)
(average rate of change on $[a, b]$)

**Instantaneous Rate of Change:** use the derivative (slope at one point)

**Definition of the Derivative:** $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

**Alternate Form of the Definition:** $f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

**Mean Value Theorem Equation:** $f'(c) = \frac{f(b)-f(a)}{b-a}$
for $a < c < b$ (for $f(x)$ continuous on [a, b] and differentiable on (a, b))




**Approximation of the Area Under a Curve**

a) **Riemann Sum:** Draw a figure and add up the rectangle areas (left, right, midpoint)
b) **Trapezoids:** (with unequal widths) add up areas of all trapezoids $A = \frac{1}{2}w(h_1 + h_2)$

(with $n$ equal widths) $A \approx \frac{1}{2}w(h_1 + 2h_2 + 2h_3 + \ldots + 2h_n + h_{n+1})$

**Differential Equations:** Separate variables and integrate.

**Approximations using a Tangent Line:** Use a convenient point to find a tangent line equation.
Plug in the given $x$-value to find an approximate $y$-value.

**Derivatives of Inverse Functions:**

For $f$ and $g$ inverse functions $g'(b) = \frac{1}{f'(a)}$ where $f(a) = b$ and $g(b) = a$.

**First Fundamental Theorem of Calculus:**

$$\int_a^b f'(x)\,dx = f(b) - f(a) \quad \text{or} \quad f(b) = f(a) + \int_a^b f'(x)\,dx$$

**Second Fundamental Theorem of Calculus:**

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Chain rule version: $\frac{d}{dx}\int_u^v f(t)\,dt = f(v)v' - f(u)u'$ (Where $u$ and $v$ are functions of $x$.)

# Memorize #2

## **Derivatives** (in order of importance)

Product rule: 1. $\frac{d}{dx}(uv) = uv' + vu'$

Quotient rule: 2. $\frac{d}{dx}\frac{t}{b} = \frac{bt' - tb'}{b^2}$

Chain rule: 3. $\frac{d}{dx}f(u) = f'(u)u'$ or $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

| | | **x form** | **u form (chain rule)** |
|---|---|---|---|
| Power Rule | 4. | $\frac{d}{dx}x^n = nx^{n-1}$ | $\frac{d}{dx}u^n = nu^{n-1}u'$ |
| Log Rule | 5. | $\frac{d}{dx}\ln x = \frac{1}{x}$ | $\frac{d}{dx}\ln u = \frac{u'}{u}$ |
| Exponential Rule | 6. | $\frac{d}{dx}e^x = e^x$ | $\frac{d}{dx}e^u = e^u u'$ |
| Trig Rules | 7. | $\frac{d}{dx}\sin x = \cos x$ | $\frac{d}{dx}\sin u = \cos u\, u'$ |
| | 8. | $\frac{d}{dx}\cos x = -\sin x$ | $\frac{d}{dx}\cos u = -\sin u\, u'$ |
| | 9. | $\frac{d}{dx}\tan x = \sec^2 x$ | $\frac{d}{dx}\tan u = \sec^2 u\, u'$ |
| Inverse Trig | 10. | $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$ | $\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$ |
| | 11. | $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$ | $\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$ |
| Exponential Rule | 12. | $\frac{d}{dx}a^x = a^x \ln a$ | $\frac{d}{dx}a^u = a^u u' \ln a$ |
| Log Rule | 13. | $\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$ | $\frac{d}{dx}\log_a u = \frac{u'}{u \ln a}$ |
| Trig Rules | 14. | $\frac{d}{dx}\cot x = -\csc^2 x$ | $\frac{d}{dx}\cot u = -\csc^2 u\, u'$ |
| | 15. | $\frac{d}{dx}\sec x = \sec x \tan x$ | $\frac{d}{dx}\sec u = \sec u \tan u\, u'$ |
| | 16. | $\frac{d}{dx}\csc x = -\csc x \cot x$ | $\frac{d}{dx}\csc u = -\csc u \cot u\, u'$ |

IMPLICIT DIFFERENTIATION can be used when it is inconvenient to solve for $y$.

**CONTINUED**

# INTEGRALS (in order of importance)

| | x form | u form (reverse chain rule) |
|---|---|---|
| Power Rule | 1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$ | $\int u^n u'\,dx = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$ |
| Log Rule | 2. $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$ | $\int \frac{u'}{u}\,dx = \ln\lvert u\rvert + C$ |
| Exponential Rule | 3. $\int e^x dx = e^x + C$ | $\int e^u u'\,dx = e^u + C$ |
| Trig Rules | 4. $\int \cos x\,dx = \sin x + C$ | $\int \cos u\, u'\,dx = \sin u + C$ |
| | 5. $\int \sin x\,dx = -\cos x + C$ | $\int \sin u\, u'\,dx = -\cos u + C$ |
| | 6. $\int \sec^2 x\,dx = \tan x + C$ | $\int \sec^2 u\, u'\,dx = \tan u + C$ |
| | 7. $\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$ | $\int \tan u\, u'\,dx = -\ln\lvert\cos u\rvert + C$ |
| Inverse Trig Rules | 8. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$ | $\int \frac{u'}{a^2+u^2}\,dx = \frac{1}{a}\arctan\frac{u}{a} + C$ |
| | 9. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$ | $\int \frac{u'}{\sqrt{a^2-u^2}}\,dx = \arcsin\frac{u}{a} + C$ |
| Trig Rule | 10. $\int \cot x\,dx = \ln\lvert\sin x\rvert + C$ | $\int \cot u\, u'\,dx = \ln\lvert\sin u\rvert + C$ |
| Exponential Rule | 11. $\int a^x dx = \frac{a^x}{\ln a} + C$ | $\int a^u u'\,dx = \frac{a^u}{\ln a} + C$ |
| Trig Rules | 12. $\int \sec x \tan x\,dx = \sec x + C$ | $\int \sec u \tan u\, u'\,dx = \sec u + C$ |
| | 13. $\int \csc^2 x\,dx = -\cot x + C$ | $\int \csc^2 u\, u'\,dx = -\cot u + C$ |
| | 14. $\int \csc x \cot x\,dx = -\csc x + C$ | $\int \csc u \cot u\, u'\,dx = -\csc u + C$ |

# Memorize #3

## Key Trigonometric and Logarithmic Facts, Formulas, and Identities

*** **SOH-CAH-TOA**

***1) Know how to get sine, cosine, tangent, cosecant, secant, and cotangent values

($\sin 0$, $\cos\frac{\pi}{6}$, $\tan\frac{\pi}{2}$, $\sec\frac{\pi}{3}$, $\csc\frac{\pi}{4}$, $\cot\frac{2\pi}{3}$, $\sin(-\pi)$, $\tan\left(-\frac{5\pi}{4}\right)$, $\cos\frac{3\pi}{2}$, etc.)

Know how to get Inverse Trig Function values (arcsin 0 , arccos 1, arctan $\sqrt{3}$, etc.)

Even Trig Functions: Cos, Sec  * $\cos(-x) = \cos x$
$\sec(-x) = \sec x$

Odd Trig Functions: Sin, Csc, Tan, Cot  * $\sin(-x) = -\sin x$
$\csc(-x) = -\csc x$
* $\tan(-x) = -\tan x$
$\cot(-x) = -\cot x$

2) Trig. Identities:

$\sin^2 x + \cos^2 x = 1$ ***

$\tan^2 x + 1 = \sec^2 x$ ** $\left(\tan^2 x = \sec^2 x - 1\right)$

$1 + \cot^2 x = \csc^2 x$ * $\left(\cot^2 x = \csc^2 x - 1\right)$

$\sin 2x = 2\sin x\cos x$ *

** $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1}{\cot x}$

* $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{\tan x}$

* $\csc x = \dfrac{1}{\sin x}$

* $\sec x = \dfrac{1}{\cos x}$

3) Original period for sin, cos, csc, sec = 2π.  Original period for tan, cot = π.
For $y = a\sin\left(b(x+c)\right) + d$ or any other trig function

** period = $\dfrac{\text{original period}}{|b|}$  * Amplitude = $|a|$ for sine and cosine

* horizontal shift = -c  * vertical shift = d

4) Definition $\log_a x = b \leftrightarrow a^b = x$ *

$f(x) = \log_a x$ and $g(x) = a^x$ are inverse functions. *

$\log_e x = \ln x$  $\ln x$ and $e^x$ are inverse functions. *

The domain of $y = \ln x$ is $x > 0$. *

The range of $y = e^x$ is $y > 0$. *

** $\ln e = 1$  ** $\ln e^x = x$  ** $e^{\ln x} = x$

** $\ln 1 = 0$  ** $e^0 = 1$

Properties: assume $a > 0$ and b > 0

* $\ln(ab) = \ln a + \ln b$

* $\ln\dfrac{a}{b} = \ln a - \ln b$

* $\ln a^k = k\ln a$

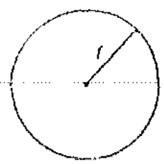
(change of base formula)

* $\log_a x = \dfrac{\ln x}{\ln a}$

# MEMORIZE #4

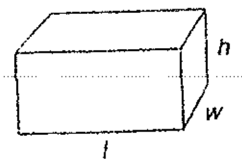
Geometry Formulas

**Circle**



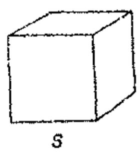

Area = $\pi r^2$
Circumference = $2\pi r$

**Rectangular Solid**



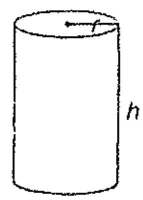

Volume = $lwh$

**Cube**



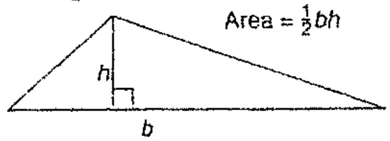

Volume = $s^3$
Surface Area = $6s^2$

**Cylinder**



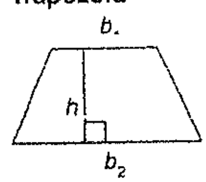

Volume = $\pi r^2 h$

**Triangle**




Area = $\frac{1}{2}bh$

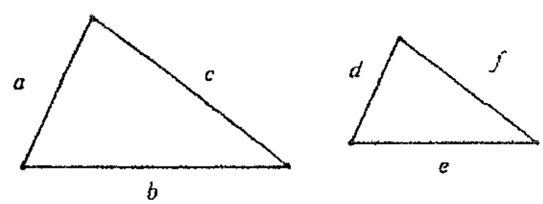
**Trapezoid**




Area = $\frac{1}{2}h(b_1 + b_2)$

Similar Triangles:
proportional sides

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



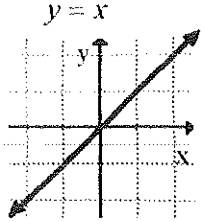

Memory Sheet #4 is continued on the next page.

## MEMORIZE #4 (continued)

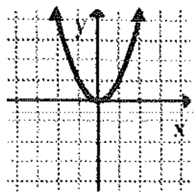
**Parent Graphs**

These graphs occur so frequently in this course that it would be worth your time to learn (memorize) them.

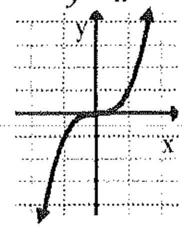
$y = x$



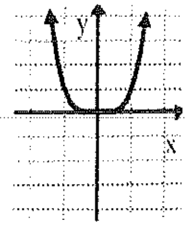

$y = x^2$



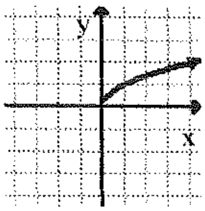

$y = x^3$



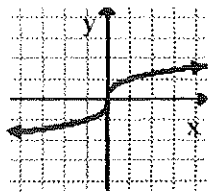

$y = x^4$



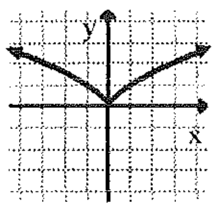

$y = \sqrt{x}$



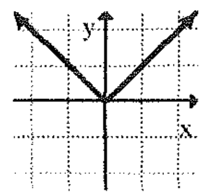

$y = \sqrt[3]{x}$



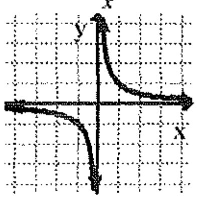

$y = x^{\frac{2}{3}}$



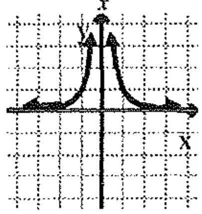

$y = |x|$



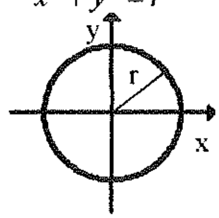

$y = \frac{1}{x}$



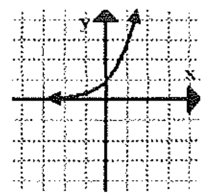

$y = \frac{1}{x^2}$



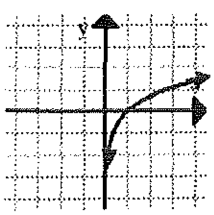

$x^2 + y^2 = r^2$



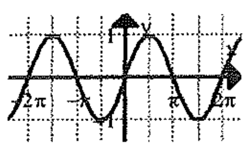

$y = e^x$



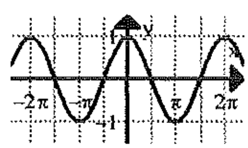

$y = \ln x$



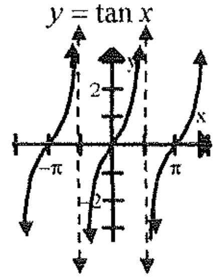

$y = \sin x$




$y = \cos x$




$y = \tan x$




Know how to find intercepts and asymptotes of graphs.

Know how to make adjustments to these graphs.