Math 12-6 (10:30 AM)—Spring, 2025 Final (**Two** hours)
D.C. Smolarski, S.J. Monday, June 9, 2025, 9:10 AM

1. (10) *(Taken from Midterm I.)* Evaluate $\int \cos^2 2x\,dx$.
2. (10) *(Taken from Midterm II.)* You are given the area bounded by the curve $y = \sec 2x$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{8}$. Rotate this area around the x-axis and find the volume of the resulting solid.
3. (10) *(Taken from Midterm III.)* Evaluate: $\int x^3 e^{x^2} dx$
4. (10) *(Taken from the Fall 2023 sample final.)* Evaluate $\int \frac{dx}{1-x^2}$ via trig substitution.
5. (12) Solve $\frac{dy}{dx} = e^{-2x}y$ given $x = 0$ and $y = 1$.
6. (12) Perform the following integration by means of partial fractions, WITHOUT evaluating the coefficients, *A, B,* ...: $\int \frac{dx}{(x^2-4)(x^2+1)}$. (I.e., you do NOT have to compute numerical values for *A, B,* ..., but merely include these coefficients in the final answer.)
7. (12) Integrate $\int x^3\sqrt{x^2-4}\,dx$.
8. (13) Evaluate $\int_{-1}^{1} \frac{1}{x^4} dx$.
9. (36) *Examine*, but DO NOT *integrate* the following 6 expressions. By *examine*, please indicate the NUMBER of the method you would use to perform the integration from the possible methods listed below. *If more than one method is possible, indicate the "best" (i.e., the simplest) method.* NOTE: you do NOT have to use a different method for each integral — this problem could be arranged so that all integrals might be done by one and the same method (unlikely though that be)!

   (a) $\int \frac{1}{x^2-x^3}\,dx$ (b) $\int \frac{1}{\sqrt{x^2-2x+1}}\,dx$ (c) $\int \ln x dx$

   (d) $\int \frac{1}{\sqrt{1-x^2}}\,dx$ (e) $\int \frac{x}{\sqrt{1-x^2}}\,dx$ (f) $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

   \# 1. SIN substitution. # 2. TAN substitution. # 3. SEC substitution.
   \# 4. partial fractions (after first factoring, if necessary).
   \# 5. complete the square, then a trig substitution (indicate which one).
   \# 6. simple substitution (indicate what u equals) and the rule after the substitution.
   \# 7. integration by parts (including the "table" version). Indicate u and dv (or $f(x)$ and $g(x)$).
   \# 8. algebraically simplify expression first, then use an elementary rule NOT listed (indicate which rule).

125 points total. P.S. Have a great summer!