Math 12-6 (10:30 AM)—Spring, 2025
D.C. Smolarski, S.J.

Midterm III (45 minutes)
Friday, May 16, 2025

NOTES:

1. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
2. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
3. **NO CALCULATORS!**

\# Perfect scores per problem

⟹ NOTE: PROBLEMS 1 AND 2 INVOLVE THE TRIANGLE (CALL IT A) FORMED BY THE LINES $y = x/3$, $x = 3$ AND $y = 2$. *You should* SKETCH *this triangle, and the appropriate figures of revolution and* INDICATE *the quantities necessary for finding the volumes, e.g.,* RADIUS *(radii) and* HEIGHT.

0/16 HARDEST

1. (15) Rotate triangle A around the line $y = 1$ to generate a solid. Set up (but do *not* integrate) the integral needed to find the volume of this solid via the method of *washers* or *disks*.

2/16

2. (15) Rotate triangle A around the y-axis to generate a solid. Set up (but do *not* integrate) the integral needed to find the volume of this solid via the method of *cylindrical shells*.

1/16

3. (12) Find the distance along the curve $y = \dfrac{2}{3}x^{3/2} - \dfrac{1}{2}x^{1/2}$ from $x = 0$ to $x = 4$.

1/16

4. (12) Evaluate: $\displaystyle\int x^3 e^{x^2} dx$

5/16

5. (12) Evaluate: $\displaystyle\int x \sin 2x dx$

7/16 EASIEST

6. (12) Evaluate: $\displaystyle\int \frac{x^3}{1+x^4} dx$

2/16

7. (12) Evaluate: $\displaystyle\int \frac{x}{1+x^4} dx$

1/16

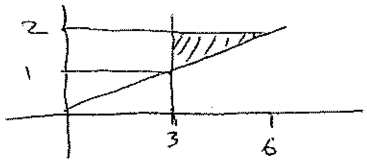
8. (10) Solve $\dfrac{dy}{dx} = \dfrac{y^3}{x}$, given $x = 1$ and $y = 2$. (Note: you do NOT have to refine the solution to a form where y is alone on one side of an equation.)

STATS

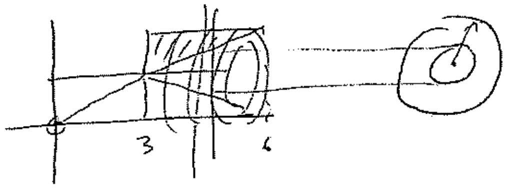
HI 87 MED ⟨49⟩ σ 15.02

LO 30 MEAN 52.75

Mt 12 - MID III - 525

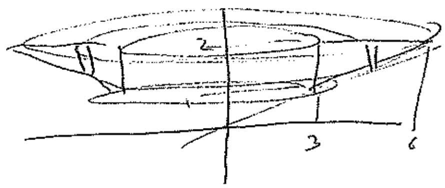


$y=2$
$x=3$
$y=\frac{x}{3}$

1. 

$r_o = 2-1=1$
$r_i = \frac{x}{3}-1$

$$V=\pi\int_3^6 1^2-\left(\frac{x}{3}-1\right)^2 dx$$

2. 

$3\ h=2-\frac{x}{3}$

$$V=2\pi\int_3^6 \overset{r}{x}\overset{h}{\left(2-\frac{x}{3}\right)}dx$$

3. $y=\frac{2}{3}x^{3/2}-\frac{1}{2}x^{1/2}$ $\quad x=0,4$

$\frac{dy}{dx}=\cancel{\frac{2}{3}}\cancel{\frac{3}{2}}x^{1/2}-\frac{1}{2}\frac{1}{2}x^{-1/2}$

$=x^{1/2}-\frac{1}{4x^{1/2}}=\frac{4x-1}{4x^{1/2}}$

$\left(\frac{dy}{dx}\right)^2=\frac{16x^2-8x+1}{16x} \Rightarrow \left(\frac{dy}{dx}\right)^2+1=\frac{16x^2-8x+1}{16x}+\frac{16x}{16x}$

$=\frac{16x^2+8x+1}{16x}=\left(\frac{4x+1}{4x^{1/2}}\right)^2$

$$L=\int_0^4\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_0^4\sqrt{\left(\frac{4x+1}{4x^{1/2}}\right)^2}\,dx=\int_0^4 x^{1/2}+\frac{1}{4}x^{-1/2}dx=\left.\frac{2x^{3/2}}{3}+\frac{1}{\cancel{4}_2}\frac{x^{1/2}}{1/2}\right]_0^4$$

$$=\frac{2}{3}4^{3/2}+\frac{1}{2}4^{1/2}-0=\frac{2}{3}\cdot 8+\frac{1}{\cancel{2}}\cancel{2}=\frac{16}{3}+1=\frac{19}{3}$$

4. $\int x^3e^{x^2}dx=\int x^2e^{x^2}x\,dx\ \left[\begin{array}{ll} w=x^2 & \\ \frac{dw}{dx}=2x & \frac{dw}{2}=x\,dx\end{array}\right]=\frac{1}{2}\int w e^w dw\ \left[\begin{array}{ll} u=w & dv=e^w dw\\ du=dw & v=e^w\end{array}\right.$

$=\frac{1}{2}\left[we^w-\int e^w dw\right]=\frac{1}{2}we^w-\frac{1}{2}e^w+C=\frac{x^2e^{x^2}}{2}-\frac{e^{x^2}}{2}+C$

5. $\int x\sin 2x\,dx\ \left[\begin{array}{ll} u=x & dv=\sin 2x\,dx\\ du=dx & v=\frac{-\cos 2x}{2}\end{array}\right]=\frac{-x\cos 2x}{2}+\frac{1}{2}\int\cos 2x\,dx$

$=\frac{-x\cos 2x}{2}+\frac{\sin 2x}{2\cdot 2}+C=\frac{-x\cos 2x}{2}+\frac{\sin 2x}{4}+C$

6. $\int\frac{x^3}{1+x^4}dx\ \left[\begin{array}{ll} u=1+x^4 & \\ \frac{du}{dx}=4x^3 & \frac{du}{4}=x^3dx\end{array}\right]=\frac{1}{4}\int\frac{du}{u}=\frac{1}{4}\ln|u|+C=\frac{\ln(1+x^4)}{4}+C$

7. $\int\frac{x}{1+x^4}dx=\int\frac{x}{1+(x^2)^2}dx\ \left[\begin{array}{ll} u=x^2 & \\ \frac{du}{dx}=2x & \frac{du}{2}=x\,dx\end{array}\right]=\frac{1}{2}\int\frac{du}{1+u^2}=\frac{1}{2}\arctan u+C$

$=\frac{1}{2}\arctan x^2+C$

8. $\frac{dy}{dx}=\frac{y^3}{x}$ $\quad\left(\begin{array}{l}x=1\\y=2\end{array}\right)$ $\quad\int\frac{dy}{y^3}=\int\frac{dx}{x}\Rightarrow\int y^{-3}dy=\ln|x|+C\Rightarrow\frac{y^{-2}}{-2}=\ln|x|+C$

$\Rightarrow\frac{-1}{2\cdot 2^2}=0+C=\frac{-1}{8}\Rightarrow\frac{-1}{2y^2}=\ln|x|-\frac{1}{8}$