Math 12-6 (10:30 AM)—Spring, 2025
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Midterm II (45 minutes)
Friday, May 2, 2025

NOTES:

perfect scores per problem

1. Make sure your NAME is on the front of the green book.
2. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
3. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
4. There are 100 points possible on this test. The point value of each problem is listed in parentheses after the number.
5. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
6. **NO CALCULATORS!**

10/16

1. (12) Once upon a time, there was a wise old man who lived in a big castle. Being as all castles are, this one was very damp and hard to heat. The old man was especially perplexed since there was a large doorway from the *calefactory* (living room) to the *refectory* (dining room) and the heat from the fire place in the calefactory was escaping into the refectory where no one every stayed for long (since the food was not very good). The doorway in question could be described by the equation $y = 9\cos x$ between $x = -\pi/2$ and $x = \pi/2$. The man, although he was very wise, never took Math 12 and was at a loss to try to find the area of this doorway (i.e., the area under the curve), since he wanted to know how much material to buy to close this passage and keep the heat in the calefactory. Please help the wise old man, and find the exact area of his doorway, rendering the final answer as a single number.

4/16

2. (15) *(Fill in the blanks:)* The RIEMANN INTEGRAL or DEFINITE INTEGRAL can be defined as the limit, as (a) $n \longrightarrow$ __________, of the (b) __________ of areas of (c) __________ whose dimensions are derived from subdividing the total area between a curve and the x-axis into vertical strips.

14/16 EASIEST

3. (12) Evaluate $\sum_{i=1}^{4}(3i^2 - 1)$.

5/16

4. (13) Find the area between the parabola $y = -x^2 + 3x - 2$ and the x-axis.

7/16

5. (12) Evaluate $\int_0^1 \frac{x^2}{(2-x^3)^2}dx$

6. (12) Evaluate $\int_1^2 \frac{(2-x^3)^2}{x^2}dx$ 3/16 HARDEST TIE

3/16 HARDEST TIE

7. (12) You are given the area bounded by the curve $y = \sec 2x$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{8}$. Rotate this area around the x-axis and find the volume of the resulting solid.

7/16

8. (12) You are given the area bounded by the curves $y = e^{3x}$ and $y = \sqrt{x}$ and the lines $x = 1$ and $x = 2$. Rotate this area around the x-axis to obtain a solid. Set up the integral to find the volume of this solid, but do NOT integrate!

STATS HI 100 MED <74> σ 14.91
LO 51 MEAN 73.44

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① $\displaystyle\int_{-\pi/2}^{\pi/2} 9\cos x\,dx = 9\sin x\Big]_{-\pi/2}^{\pi/2} = 9\sin\frac{\pi}{2} - 9\sin\left(-\frac{\pi}{2}\right) = 9\cdot 1 - 9(-1) = 9+9 = 18$

② RIEMANN INTEGRAL is the limit, as (a) $\underline{n \to \infty}$ (or $\underline{\Delta x \to 0}$) of the (b) <u>SUM</u> of areas of (c) <u>RECTANGLES</u> whose dimensions are derived from subdividing the total area into vertical strips

③
$$\begin{aligned}\sum_{i=1}^{4}(3i^2-1) &= 3\cdot 1^2-1 + 3\cdot 2^2-1 + 3\cdot 3^2-1 + 3\cdot 4^2-1\\ &= 3\cdot 1-1 + 3\cdot 4-1 + 3\cdot 9-1 + 3\cdot 16-1\\ &= 3-1 + 12-1 + 27-1 + 48-1\\ &= 2 + 11 + 26 + 47 = \underline{86}\end{aligned}$$

④ $y = -x^2+3x-2 = -(x^2-3x+2) = -(x-1)(x-2) \Rightarrow x = 1, 2$ are pts of intersection

$$A = \int_{+1}^{+2} -x^2+3x-2\,dx = \frac{-x^3}{3}+\frac{3x^2}{2}-2x\Big]_1^2 = \frac{-8}{3}+\frac{3\cdot 4}{2}-4-\left(-\frac{1}{3}+\frac{3}{2}-2\right)$$

$$= -\frac{7}{3}+6-4-\frac{3}{2}+2 = 4-\frac{7}{3}-\frac{3}{2} = \frac{24-14-9}{6} = \underline{\frac{1}{6}}$$

⑤
$$\int_0^1 \frac{x^2}{(2-x^3)^2}\,dx \quad \begin{bmatrix} u = 2-x^3 \\ \frac{du}{dx} = -3x^2 \\ \frac{du}{-3} = x^2\,dx \end{bmatrix} = -\frac{1}{3}\int_{x=0}^{1} u^{-2}\,du = -\frac{1}{3}\frac{u^{-1}}{-1}\Big]_{x=0}^{1} = \frac{1}{3(2-x^3)}\Big]_0^1$$

$$= \frac{1}{3(2-1)} - \frac{1}{3(2-0)} = \frac{1}{3}-\frac{1}{6} = \underline{\frac{1}{6}}$$

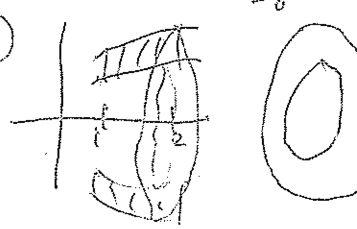
⑥
$$\int_1^2 \frac{(2-x^3)^2}{x^2}\,dx = \int \frac{4-4x^3+x^6}{x^2}\,dx = \int 4x^{-2}-4x+x^4\,dx = \frac{4x^{-1}}{-1}-\frac{4x^2}{2}+\frac{x^5}{5}\Big]_1^2$$

$$= -\frac{4}{x}-2x^2+\frac{x^5}{5}\Big]_1^2 = -\frac{4}{2}-2\cdot 4+\frac{32}{5}-\left(-\frac{4}{1}-2+\frac{1}{5}\right) = \cancel{-2}-8+\frac{31}{5}+4+\cancel{2}$$

$$= -4+\frac{31}{5} = \frac{31-20}{5} = \underline{\frac{11}{5}}$$

⑦
$$V = \pi\int_0^{\pi/8} (\sec 2x)^2\,dx = \begin{bmatrix} u = 2x \\ \frac{du}{dx} = 2 \\ \frac{du}{2} = dx \end{bmatrix} = \frac{\pi}{2}\int_{x=0}^{\pi/8} \sec^2 u\,du = \frac{\pi}{2}\tan u\Big]_{x=0}^{\pi/8}$$

$$= \frac{\pi}{2}\tan 2x\Big]_0^{\pi/8} = \frac{\pi}{2}\left(\tan 2\left(\frac{\pi}{8}\right)-\tan 0\right) = \frac{\pi}{2}\tan\frac{\pi}{4} = \underline{\frac{\pi}{2}}$$

⑧ 

$r_o = e^{3x}$
$r_i = \sqrt{x}$ $\therefore$ $V = \pi\displaystyle\int_1^2 e^{6x} - x\,dx$