Math 12-6 (10:30 AM) — Spring, 2025 | Midterm I (45 minutes)
D.C. Smolarski, S.J. | Wednesday, April 16, 2025

NOTES:

1. Make sure your NAME is on the front of the green book.
2. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
3. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
4. There are 100 points possible on this test. The point value of each problem is listed in parentheses after the number.
5. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
6. **NO CALCULATORS!**

perfect scores per problem

1. (18) Let $f(x) = 6 - 3x^2 - x^3$.

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(a) Using Calculus methods, find all possible values of x where $f(x)$ attains a relative maximum or minimum and indicate which x-value corresponds to which type of extremal point.

(b) At which x value does $f(x)$ have a "point of inflection" (i.e., where the concavity changes)?

0 HARDEST

2. (18) Evaluate the following limits:

(a) $\lim_{x\to 0} \dfrac{\cos(x^2) - 1}{x^3}$ (b) $\lim_{x\to 0^+} \dfrac{1}{x} - \dfrac{1}{\sqrt{x}}$

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3. (14) Solve the following differential equation, given that the curve goes through the point (1,3):

$$\frac{dy}{dx} = 5x^2 - 2$$

In problems 4–7, integrate the indicated expression.

3/16 4. (10) $\int \cos^2 2x\,dx$ 8/16 5. (10) $\int \sec^2 4x\,dx$ 6/16 6. (10) $\int \dfrac{1}{\sqrt{1-4x^2}}dx$

3/16 7. (10) $\int \dfrac{x}{\sqrt{1-4x^2}}dx$

8/16 8. (10) Let $f(x) = 15x - 3x^2$. Given the interval $(a,b) = (1,3)$, find the point c in the interval that satisfies the Mean Value Theorem (i.e., that the tangent line at c is parallel to the chord from $(a, f(a))$ to $(b, f(b))$).

STATS

HI 92 MED <63> σ 17.21

LO 35 MEAN 61.06

Mt 12 S25 MID I

1 a) $f(x) = 6 - 3x^2 - x^3 \Rightarrow f'(x) = -6x - 3x^2 = -3x(2+x)$

$f'(x) = 0 \Rightarrow -3x(2+x) = 0 \Rightarrow \underline{x=0}$ or $2+x=0 \Rightarrow \underline{x=-2}$

$\therefore$ max/min at $x = 0$ or -2.

$f''(x) = -6 - 6x \Rightarrow f''(0) = -6 - 0 < 0 \therefore \underline{x=0 \text{ is a max.}}$ (conc. down)

$f''(-2) = -6 + 12 = 6 > 0 \therefore \underline{x=-2 \text{ is a min}}$ (conc. up.)

b) $f''(x) = -6 - 6x = 0 \Rightarrow 6x = -6 \Rightarrow \underline{x = \frac{-6}{6} = -1}$ $\in$ pt of inflection.

2 a) $\lim_{x\to 0} \frac{\cos(x^2)-1}{x^3} \overset{L'H}{=} \lim \frac{-\sin(x^2)\cdot 2x}{3x^2} \overset{L'H}{=} \lim \frac{-\sin(x^2)\cdot 2 + 2x(-\cos x^2)2x}{6x}$

$\overset{L'H}{=} \lim \frac{-2\cos(x^2)2x + 4x^2(\sin x^2)2x - (\cos x^2)8x}{6} = \frac{0}{6} = \underline{0}$ $\left[\text{or } = \lim \frac{-\sin(x^2)2}{3x} = \lim \frac{-2\cos(x^2)2x}{3} = 0\right.$

b) $\lim_{x\to 0^+} \frac{1}{x} - \frac{1}{\sqrt{x}} = \lim \frac{1 - \sqrt{x}^{\to 0}}{x_{\to 0}} = \underline{\infty}$

3. $\frac{dy}{dx} = 5x^2 - 2 \quad (1,3) \quad \Rightarrow \int dy = \int 5x^2 - 2\, dx \Rightarrow y = \frac{5x^3}{3} - 2x + C$

$\Rightarrow 3 = \frac{5}{3} - 2 + C \Rightarrow C = 3 + 2 - \frac{5}{3} = \frac{15-5}{3} = \frac{10}{3} \therefore \underline{y = \frac{5x^3}{3} - 2x + \frac{10}{3}}$

4 $\int \cos^2 2x\, dx = \frac{1}{2}\int 1 + \cos 4x\, dx = \frac{1}{2}x + \frac{1}{2}\frac{\sin 4x}{4} + C = \underline{\frac{x}{2} + \frac{\sin 4x}{8} + C}$

5 $\int \sec^2 4x\, dx \begin{bmatrix} u = 4x \\ du = 4dx \\ \frac{du}{4} = dx \end{bmatrix} = \frac{1}{4}\int \sec^2 u\, du = \frac{1}{4}\tan u + C = \underline{\frac{\tan 4x}{4} + C}$

6. $\int \frac{dx}{\sqrt{1-4x^2}} \begin{bmatrix} u = 2x \\ du = 2dx \\ \frac{du}{2} = dx \end{bmatrix} = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin u + C = \underline{\frac{\arcsin 2x}{2} + C}$

7. $\int \frac{x\, dx}{\sqrt{1-4x^2}} \begin{bmatrix} u = 1-4x^2 \\ du = -8x\, dx \\ \frac{-du}{8} = x\, dx \end{bmatrix} = -\frac{1}{8}\int \frac{du}{\sqrt{u}} = -\frac{1}{8}\int u^{-1/2} du = \frac{-1}{\frac{8}{4}} \frac{u^{1/2}}{1/2} + C$

$= \underline{-\frac{1}{4}\sqrt{1-4x^2} + C}$

8. $f(x) = 15x - 3x^2 \quad f'(x) = 15 - 6x$

$(1,3)$

$\frac{f(3) - f(1)}{3-1} = f'(c) = 15 - 6c$

$\frac{45 - 27 - (15-3)}{2} = \frac{30-24}{2} = \frac{6}{2} = 3$

$= 15 - 6c \Rightarrow 3 - 15 = -6c$

$\Rightarrow 12 = 6c \Rightarrow c = \frac{12}{6} = \underline{2}$