Math 12-7 (1:00 PM)—Fall, 2023
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Midterm III (45 minutes)
Friday, November 3, 2023

perfect scores per problem

NOTES:

1. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
2. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
3. **NO CALCULATORS!**

$\Longrightarrow$NOTE: PROBLEMS 1 AND 2 INVOLVE THE TRIANGLE (CALL IT A) FORMED BY THE LINES $y = x$, $x = 2$ AND $y = 1$. *You should sketch this triangle, and the appropriate figures of revolution and indicate the quantities necessary for finding the volumes, e.g., radius (radii) and height.*

23/35 1. (14) Rotate triangle A around the line $y = 1$ to generate a solid. Set up (but do *not* integrate) the integral needed to find the volume of this solid via the method of *washers* or *disks*.

14/35 2. (14) Rotate triangle A around the y-axis to generate a solid. Set up (but do *not* integrate) the integral needed to find the volume of this solid via the method of *cylindrical shells*.

15/35 3. (14) The area between the lines $y = e^{2x}$, $x = 1$, and $x = 2$ is rotated around the x-axis. Find the volume of the resulting solid.

5/35 HARDEST 4. (14) Find the distance along the curve $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$ from $x = 1$ to $x = 2$.

10/35 5. (10) Evaluate $\int 4^{2x}\,dx$.

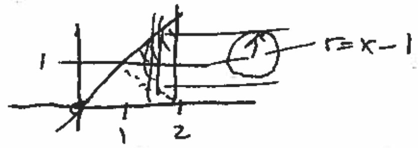
10/35 6. (12) Evaluate $\int x \sec^2 3x\,dx$.

20/35 7. (10) Evaluate $\int \dfrac{3x}{\sec 3x}\,dx$.

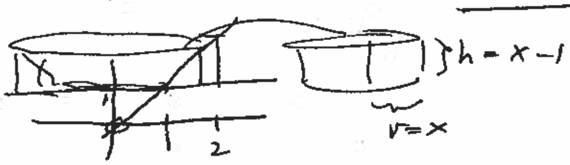
24/35 EASIEST 8. (12) Solve $\dfrac{dy}{dx} = \dfrac{y}{x^2}$ given $x = 2$ and $y = 1$.

STATS HI 100 MED 70 σ 17.65
LO 38 MEAN 69.4 # EXAMS 35

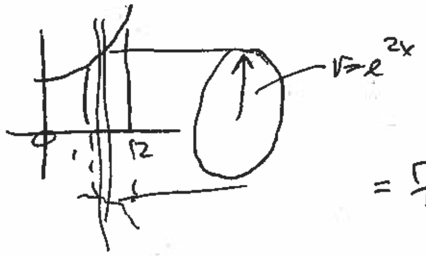
Mt 12 – MID III – F23

1. 

$$V=\pi\int_1^2 (x-1)^2\,dx$$

2. 

$$V=2\pi\int_1^2 x(x-1)\,dx$$

3. 

$$V=\pi\int_1^2 \left(e^{2x}\right)^2 dx = \pi\int e^{4x}\,dx \quad \begin{bmatrix} u=4x & \frac{du}{4}=dx \\ \frac{du}{dx}=4 & \end{bmatrix}$$

$$=\frac{\pi}{4}\int_{x=1}^{2} e^u\,du = \frac{\pi}{4}e^u\Big]_{x=1}^{2} = \frac{\pi}{4}e^{4x}\Big]_1^2 = \frac{\pi}{4}\left(e^8-e^4\right)$$

4. $$y=\frac{x^4}{4}+\frac{1}{8x^2}=\frac{x^4}{4}+\frac{1}{8}x^{-2} \Rightarrow \frac{dy}{dx}=\frac{\cancel{4}x^3}{\cancel{4}}-\frac{2}{8}x^{-3}=x^3-\frac{1}{4x^3}=\frac{4x^6-1}{4x^3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2=\frac{16x^{12}-8x^6+1}{16x^6} \Rightarrow \left(\frac{dy}{dx}\right)^2+1=\frac{16x^{12}-8x^6+1}{16x^6}+\frac{16x^6}{16x^6}=\frac{16x^{12}+8x^6+1}{16x^6}=\left(\frac{4x^6+1}{4x^3}\right)^2$$

$$\Rightarrow L=\int_1^2\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int\sqrt{\left(\frac{4x^6+1}{4x^3}\right)^2}\,dx=\int x^3+\frac{1}{4}x^{-3}\,dx=\frac{x^4}{4}+\frac{1}{4}\,\frac{x^{-2}}{-2}\Big]_1^2$$

$$=\frac{16}{4}-\frac{1}{32}-\left(\frac{1}{4}-\frac{1}{8}\right)=4-\frac{1}{32}-\frac{1}{8}=\frac{128-1-4}{32}=\frac{123}{32}=3\tfrac{27}{32}$$

5. $$\int 4^{2x}\,dx \quad \begin{bmatrix} u=4^{2x} \\ \ln u=\ln(4^{2x})=2x\ln 4 \\ \frac{1}{u}\frac{du}{dx}=2\ln 4 \\ \frac{du}{2\ln 4}=u\,dx=4^{2x}dx \end{bmatrix} = \frac{1}{2\ln 4}\int du=\frac{u}{2\ln 4}+C=\frac{4^{2x}}{2\ln 4}+C$$

6. $$\int x\sec^2 3x\,dx \quad \begin{bmatrix} u=x & dv=\sec^2 3x\,dx \\ du=dx & v=\frac{\tan 3x}{3} \end{bmatrix} = \frac{x\tan 3x}{3}-\frac{1}{3}\int\tan 3x\,dx$$

$$=\frac{x\tan 3x}{3}-\frac{1}{3}\int\frac{\sin 3x}{\cos 3x}\,dx. \quad \begin{bmatrix} w=\cos 3x \\ \frac{dw}{dx}=-3\sin 3x \\ -\frac{dw}{3}=\sin 3x\,dx \end{bmatrix} = \frac{x\tan 3x}{3}-\frac{1}{3}\left(-\frac{1}{3}\right)\int\frac{dw}{w}$$

$$=\frac{x\tan 3x}{3}+\frac{1}{9}\ln|w|+C = \frac{x\tan 3x}{3}+\frac{1}{9}\ln|\cos 3x|+C$$

7. $$\int\frac{3x}{\sec 3x}\,dx=\int 3x\cos 3x\,dx \quad \begin{bmatrix} u=x & dv=\cos 3x\,dx \\ du=dx & v=\frac{\sin 3x}{3} \end{bmatrix} = 3\left[\frac{x\sin 3x}{3}-\frac{1}{3}\int\sin 3x\,dx\right]$$

$$=x\sin 3x+\frac{\cos 3x}{3}+C$$

8. $$\frac{dy}{dx}=\frac{y}{x^2} \Rightarrow \int\frac{dy}{y}=\int x^{-2}dx \Rightarrow \ln|y|=\frac{x^{-1}}{-1}+C\,\Big|_{(2,1)} \Rightarrow \cancelto{0}{\ln 1}=-\frac{1}{2}+C$$

$$\Rightarrow C=\frac{1}{2} \Rightarrow \ln|y|=\frac{1}{2}-\frac{1}{x}$$