Math 12-7 (1:00 MM)—Fall, 2023 | Midterm II (45 minutes)
D.C. Smolarski, S.J. | Friday, October 20, 2023

NOTES:

1. Make sure your NAME is on the front of the blue book.
2. Problems can be written in the blue book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
3. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
4. There are 100 points possible on this test. The point value of each problem is listed in parentheses after the number.
5. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
6. **NO CALCULATORS!**

perfect scores per problem

22/35 "EASIEST

1. (8) Show that these two sums either do evaluate to the same number or don't.
$$\sum_{k=2}^{4} \frac{(-1)^{k-1}}{k-1} \text{ and } \sum_{k=-1}^{1} \frac{(-1)^k}{k+2}.$$

19/35

2. (12) Once upon a time, there was a wise old man who lived in a big castle. Being as all castles are, this one was very damp and hard to heat. The old man was especially perplexed since there was a large doorway from the *calefactory* (living room) to the *refectory* (dining room) and the heat from the fire place in the calefactory was escaping into the refectory where no one every stayed for long (since the food was not very good). The doorway in question could be described by the equation $y = (x^2 - 4)^2$ between -2 and 2. The man, although he was very wise, never took Math 12 and was at a loss to try to find the area of this doorway (i.e., the area under the curve), since he wanted to know how much material to buy to close this passage and keep the heat in the calefactory. Please help the wise old man, and find the exact area of his doorway.

17/35

3. (10) Find $\frac{dF}{dx}$ when $F(x) = \int_2^x 3^t dt$.

13/35 "HARDEST"

4. (10) Choose the correct answer to complete the statement: *The Fundamental Theorem of the Calculus says that ... :*
   (a) indefinite integrals can be evaluated via definite integrals.
   (b) definite integrals can be evaluated via indefinite integrals.
   (c) area under a curve is the limit of the sum of rectangle areas.

17/35

5. (12) Find the *total area* between the graph of the curve $y = x^3 - 3x^2 + 2x$ and the x-axis.

20/35

6. (12) Evaluate $\displaystyle\int_2^3 \frac{x}{(1-x^2)^2} dx$

19/35

7. (12) Evaluate $\displaystyle\int_2^3 \frac{(1-x^2)^2}{x} dx$

15/35

8. (12) You are given the area bounded by the curve $y = 2x - x^2$ and the x-axis. Rotate this area around the x-axis and find the volume of the resulting solid.

21/35

9. (12) You are given the area bounded by curve $y = 2 - x^2$ and the line $y = 1$. Rotate this area around the x-axis to obtain a solid. Set up the integral to find the volume of the resulting solid but do NOT integrate!

STATS HI 97 MEDIAN 83 σ 12.84
LO 43 MEAN 80.34 # EXAMS 35

M+12 – F23 – MID II

1. $\sum_{2}^{4} \frac{(-1)^{k-1}}{k-1} = \frac{(-1)^1}{1} + \frac{(-1)^2}{2} + \frac{(-1)^3}{3} = -1 + \frac{1}{2} - \frac{1}{3} = \frac{-6+3-2}{6} = \frac{-5}{6}$

$\sum_{-1}^{1} \frac{(-1)^k}{k+2} = \frac{(-1)^{-1}}{1} + \frac{(-1)^0}{2} + \frac{(-1)^1}{3} = -1 + \frac{1}{2} - \frac{1}{3} = \frac{-5}{6}$ (=)

2. Area $= \int_{-2}^{2} (x^2-4)^2 dx = \int_{-2}^{2} x^4 - 8x^2 + 16\, dx = \frac{x^5}{5} - \frac{8x^3}{3} + 16x \Big]_{-2}^{2}$

$= \frac{2^5}{5} - \frac{8\cdot 2^3}{3} + 16\cdot 2 - \left(\frac{(-2)^5}{5} - \frac{8(-2)^3}{3} + 16(-2)\right) = \frac{32}{5} - \frac{64}{3} + 32 + \frac{32}{5} - \frac{64}{3} + 32$

$= \frac{64}{5} - \frac{128}{3} + 64 = \frac{3\cdot 64 - 5\cdot 128 + 15\cdot 64}{15}$ $\left[\begin{array}{ccc} 128 & 64 & 1152 \\ 5 & 18 & -640 \\ 640 & 512 & 512 \\ & 64 & \\ & 1152 & \end{array}\right]$ $= \frac{512}{15}$

3. $F(x) = \int_{2}^{x} 3^t dt \Rightarrow \frac{dF}{dx} = 3^x$

4. Fund Thm. of Calc says
(b) def integrals can be evaluated via indef integrals

5. $y = x^3 - 3x^2 + 2x = x(x^2-3x+2) = x(x-2)(x-1) \Rightarrow x = 0,1,2$ are roots

$\int_0^1 x^3 - 3x^2 + 2x\, dx = \frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2} \Big]_0^1 = \frac{1}{4} - \cancel{1} + \cancel{1} - (0) = \frac{1}{4}$

$\int_1^2 x^3 - 3x^2 + 2x\, dx = \frac{x^4}{4} - x^3 + x^2 \Big]_1^2 = \frac{2^4}{4} - 8 + 4 - \left(\frac{1}{4} - \cancel{1} + \cancel{1}\right) = \cancel{4 - 8 + 4} - \frac{1}{4} = -\frac{1}{4}$

total area $= \left|\frac{1}{4}\right| + \left|-\frac{1}{4}\right| = \frac{1}{2}$

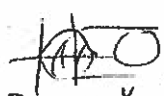
6. $\int_2^3 \frac{x}{(1-x^2)^2} dx$ $\left[\begin{array}{l} u = 1-x^2 \\ du/dx = -2x \\ -du/2 = x\,dx \end{array}\right] = -\frac{1}{2}\int_{x=2}^{3} u^{-2} du = -\frac{1}{2}\frac{u^{-1}}{-1}\Big]_{x=2}^{3} = \frac{1}{2(1-x^2)}\Big|_2^3$

$= \frac{1}{2(1-9)} - \frac{1}{2(1-4)} = -\frac{1}{16} + \frac{1}{6} = \frac{-3+8}{48} = \frac{5}{48}$

7. $\int_2^3 \frac{(1-x^2)^2}{x} dx = \int_2^3 \frac{1-2x^2+x^4}{x} dx = \int_2^3 \frac{1}{x} - 2x + x^3 dx = \ln|x| - \frac{2x^2}{2} + \frac{x^4}{4}\Big]_2^3$

$= \ln 3 - 9 + \frac{81}{4} - \ln 2 + 4 - \frac{16}{4} = \ln\frac{3}{2} - 9 + \frac{81}{4} = \ln\frac{3}{2} + \frac{81-36}{4} = \ln 3/2 + \frac{45}{4}$

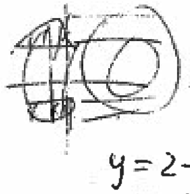
8. $y = 2x - x^2 = x(2-x) \Rightarrow x = 0, 2$ are roots



$V = \pi\int_0^2 (2x-x^2)^2 dx = \pi\int_0^2 4x^2 - 4x^3 + x^4 dx = \pi\left[\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}\right]_0^2$

$= \pi\left[\frac{4\cdot 8}{3} - 16 + \frac{32}{5} - 0\right] = \pi\left[\frac{160 - 16\cdot 15 + 96}{15}\right] = \left[\begin{array}{r} 16 \\ 15 \\ 80 \\ 16 \\ 240 \end{array}\right] \frac{(256-240)}{15}\pi = \frac{16}{15}\pi$

9. $y = 2 - x^2$ and $y = 1 \Rightarrow 1 = 2 - x^2 \Rightarrow 0 = 1 - x^2$ — $(1-x)(1+x) = 0 \Rightarrow$ intercept at $x = -1, 1$



$V = \pi\int_{-1}^{1} (2-x^2)^2 - 1^2 dx = \pi\int_{-1}^{1} 4 - 4x^2 + x^4 - 1\, dx = \pi\int_{-1}^{1} 3 - 4x^2 + x^4 dx$

$y = 2 - x^2$ ($= 2$ at $x = 0$) outer curve
$y = 1$ always — inner curve