Math 12–7 (1:00 PM)—Fall, 2023
D.C. Smolarski, S.J.

Midterm I (45 minutes)
Friday, October 6, 2023

NOTES:

1. Make sure your NAME is on the front of the green book.
2. Problems can be written in the green book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
3. PLEASE *label* (or underline or box in) all ANSWERS clearly.
4. There are 100 points possible on this test. The point value of each problem is listed in parentheses after the number.
5. Show your WORK — partial credit is possible only when all work needed to obtain an answer is presented legibly.
6. **NO CALCULATORS!**

#perfect scores per problem

26/35 1. (10) Let $f(x) = -4+5x-x^2$. Given the interval $(a,b) = (1,4)$, find the point c in the interval that satisfies Rolle's Theorem (i.e., that the tangent line at c is parallel to x-axis).

16/35 2. (18) Let $f(x) = 4x^3 + 8x^2$.

(a) Using Calculus methods, find all possible values of x where $f(x)$ attains a relative maximum or minimum and, using the second derivative, indicate which x-value corresponds to which type of extremal point.

(b) At which x value does $f(x)$ have a "point of inflection" (i.e., where the concavity changes)?

6/35 "HARDEST"

3. (18) Evaluate the following limits:

(a) $\lim_{n\to\infty} \sqrt{n^2+1} - n$ (b) $\lim_{x\to 0^+} \dfrac{x\sin x - x^2}{x^3 + x^2}$

28/35 4. (14) Solve the following differential equation, given that the curve goes through the point (1,1):

$$\frac{dy}{dx} = 3(x+1)^2$$

In problems 5–8, integrate the indicated expression.

10/35 5. (10) $\int \sin^2 3x\,dx$

29/35 "EASIEST" 6. (10) $\int xe^{2x^2}\,dx$

23/35 7. (10) $\int \dfrac{x}{1+9x^2}\,dx$

17/35 8. (10) $\int \dfrac{1}{1+9x^2}\,dx$

STATS

H1	100	MEDIAN	76	σ	17.39
LO	37	MEAN	75.2	# EXAMS	35

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1. $f(x) = -4+5x-x^2 \Rightarrow f'(x) = 5-2x = 0 \Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$

2. $f(x) = 4x^3 + 8x^2 \Rightarrow f'(x) = 12x^2 + 16x = 4x(3x+4)$

a) $f'(x) = 0 \Rightarrow x = 0$ or $3x+4=0 \Rightarrow 3x = -4 \Rightarrow x = -4/3$

$f''(x) = 24x+16 \quad f''(0) = 16 > 0 \therefore$ concave up $\therefore$ min at $x=0$

$f''(-4/3) = 24(-4/3)+16 = -32+16 = -16 < 0 \therefore$ concave down

$\therefore$ max at $x = -4/3$

b) $f''(x) = 24x+16 = 0 \Rightarrow 24x = -16 \Rightarrow x = \frac{-16}{24} = \frac{-2}{3}$ pt of inflection

3. a) $\lim_{n\to\infty} \sqrt{n^2+1} - n = \lim (\sqrt{n^2+1} - n)\frac{(\sqrt{n^2+1}+n)}{(\sqrt{n^2+1}+n)} = \lim \frac{n^2+1-n^2}{\sqrt{n^2+1}+n}$

$= \lim \frac{1}{\sqrt{n^2+1}+n} = \left(\frac{1}{\infty}\right) 0$

b) $\lim_{n\to 0} \frac{x\sin x - x^2}{x^3+x^2} \overset{L'H}{=} \lim \frac{x\cos x + \sin x - 2x}{3x^2+2x} \overset{L'H}{=} \lim \frac{-x\sin x + \cos x + \cos x - 2}{6x+2}$ ($\sin x \to 0$, $\cos x \to 1$, $6x \to 0$)

$= \frac{0}{2} = 0$

or $= \lim \frac{x(\sin x - x)}{x(x^2+x)} \overset{L'H}{=} \lim \frac{\cos x - 1}{2x+1} = \frac{0}{1} = 0$ ($\cos x \to 1$, $2x \to 0$)

4. $\frac{dy}{dx} = 3(x+1)^2 \quad (1,1) \Rightarrow \int dy = 3\int x^2+2x+1\,dx \Rightarrow y = \frac{3x^3}{3} + 3x^2 + 3x + C$

at $(1,1) \Rightarrow 1 = 1+3+3+C \Rightarrow 1 = 7+C \Rightarrow C = -6 \Rightarrow y = x^3+3x^2+3x-6$

5 $\int \sin^2 3x\,dx = \frac{1}{2}\int 1-\cos 6x\,dx = \frac{1}{2}\left[x - \frac{\sin 6x}{6}\right] + C$

6. $\int x e^{2x^2} dx \quad \left[\begin{matrix} u = 2x^2 \\ \frac{du}{4} = x\,dx \end{matrix}\right] = \frac{1}{4}\int e^u du = \frac{1}{4}e^u + C = \frac{e^{2x^2}}{4} + C$

7. $\int \frac{x}{1+9x^2} dx \quad \left[\begin{matrix} u = 1+9x^2 \\ \frac{du}{18} = x\,dx \end{matrix}\right] = \frac{1}{18}\int \frac{du}{u} = \frac{1}{18}\ln|u| + C = \frac{1}{18}\ln|1+9x^2| + C$

8. $\int \frac{1}{1+9x^2} dx \quad \left[\begin{matrix} u = 3x \\ \frac{du}{3} = dx \end{matrix}\right] = \frac{1}{3}\int \frac{du}{1+u^2} = \frac{1}{3}\arctan u + C = \frac{1}{3}\arctan 3x + C$