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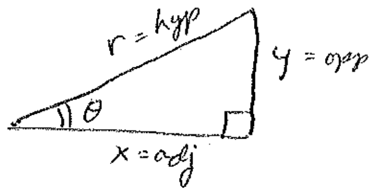
TRIG REVIEW (1,3)



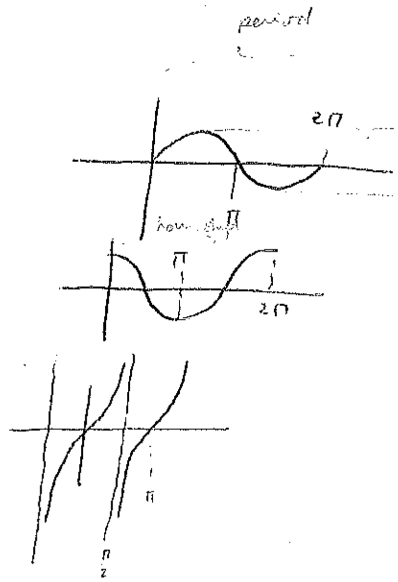
p 22

Standard functions

$$\sin\theta = \frac{y}{r} = \frac{opp}{hyp}$$

$$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$$

$$\tan\theta = \frac{y}{x} = \frac{opp}{adj}$$



Note: sin is always written of some angle.
There are no "free-floating" trig functions just like there are no "free-floating" square roots or cubes, etc.
One doesn't write $\sqrt{}$ or ^{3}. One should not write sin or cos but $\sin\theta$ or $\cos A$, etc.!

reciprocal functions

$$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$$

P24-25

Pythagorean Thm

$$x^2+y^2=r^2$$

divide by r^2

$$\left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=\left(\frac{r}{r}\right)^2 = \underline{\sin^2\theta+\cos^2\theta=1}$$

divide by $\cos^2\theta$

$$\underline{\tan^2\theta+1=\sec^2\theta}$$

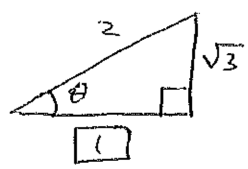
divide by $\sin^2\theta$

$$\underline{1+\cot^2\theta=\csc^2\theta}$$

Evaluating other Trig functions

If we are given one trig function and its value, using the definitions and/or Pythagorean Thm, we can get all the others.

$\sin\theta=\frac{\sqrt{3}}{2}$ $\Rightarrow$



$\Rightarrow$ $2^2=x^2+(\sqrt{3})^2$
$4=x^2+3$
$1=x^2$
$x=1$

$\therefore$ $\cos\theta=\frac{1}{2}$

$\tan\theta=\sqrt{3}$

$\sec\theta=2$

$\csc\theta=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\cot\theta=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

Radian Measure

p21 - It is relatively arbitrary how we measure a full circle or its parts. Historically, we have been given the number 360 and told there are 360 "degrees" in a full circle.

- A common alternative is called radian measure. With this measure a full circle is the circumference of a circle with radius 1, i.e. $\underline{2\pi}$.
- One can translate by comparing quantities to the full circle.

i.e. $360° = 2\pi$ rad.

$$\frac{x°}{360} = \frac{y}{2\pi} \text{ radians}$$

EG.

$90° = ?$ radians.

$$\frac{90°}{360} = \frac{1}{4} = \frac{y}{2\pi} \Rightarrow \frac{\cancel{2}\pi}{\cancel{4}_2} = y \Rightarrow \frac{\pi}{2} \text{ rad.}$$

Note: many math/engineering formulas assume the angle is given in radians!

PS-22 assume radian measure in book.

Periods

p24 A function is said to be periodic if its range values repeat as the domain is shifted, i.e. if $f(x+c) = f(x)$ for some c.

Trig functions are periodic of period 2π (i.e. 1 circle in radian measure).

$$\therefore \sin\theta = \sin(\theta + 2\pi)$$
$$\cos\theta = \cos(\theta + 2\pi)$$

EG. What is $\sin\frac{9\pi}{4}$? $\qquad \frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$

$$\sin\frac{9\pi}{4} = \sin\frac{\pi}{4} = \underline{\underline{\frac{\sqrt{2}}{2}}}.$$

Commonly used formulas

p24 Margin

① negative angle

$$\sin(-\theta) = -\sin\theta \qquad \text{(odd function)}$$

$$\cos(-\theta) = \cos\theta \qquad \text{(even function)}$$

p25

② sum / difference of 2 angles

for derivation
See Ap. B
pp ~~A-29 – A-31~~
A-14

See Ap. 4
p A-14

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

p25 ③ Double angles

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

p25 ④ Half angle

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

from these we can get

$$\lim_{h \to 0} \frac{1-\cos h}{h} = 0$$

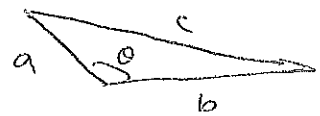
⑤ other

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

Law of Cosines

14th p25-26

basically an extentcion of the Pythoreen thm if you do not have any right angle.



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

when θ is ∡ opposite side c

Note: if $\theta = 90°$, $\cos\theta = 0$ and we get

$c^2 = a^2 + b^2$, the Pythagorean thm.

INVERSE FUNCTIONS ~~(6.1)~~ 1, 6 p11 44 p38

17-4
18-1
~~(8)~~
~~(3)~~
(93-10)

Some functions have an inverse function, but not all. These function take the output of the first function & give you back the input that gave it.

E.G. $f(x) = x^3 + 2$

$g(u) = (u-2)^{1/3}$

f and g are inverses of each other.

$f(2) = 2^3 + 2 = 8 + 2 = 10$

$g(10) = (10-2)^{1/3} = 8^{1/3} = 2$

$\Longrightarrow$ We can write $g = f^{(-1)}$

If a function can give the same <u>output</u> for different <u>inputs</u>, it has no easy inverse. We must "restrict" it.

E.G. $y = x^2$ — both $x=2$ and $x=-2$ give $y=4$

~~[illegible]~~ inverse for restricted function, i.e. $x \geq 0$ is $x = \sqrt{y}$ for $x \geq 0$ and $y \geq 0$.

If a function is such that the output values are never the same if the input values are different, it is called 1-1, e.g. $y = x$ or $y = x^3$.

$y = x^2$ is <u>NOT</u> 1-1.

1-1 functions have inverses without needing to be restricted.

(1,6) p44 14"

INVERSE TRIG FUNCTIONS

~~We first looked at inverse functions in sec 6.1~~

Look at an algebraic example again.

Let $y = \frac{1}{2}x - 5 = f(x)$

If $x = 2$ we get $y = -4$

If $x = -4$ we get $y = -7$

To get the inverse, solve for the independent variable, x. Then we get a function of y.

$$y = \frac{1}{2}x - 5$$

$$\Rightarrow \quad y + 5 = \frac{1}{2}x$$

$$\Rightarrow \quad x = 2(y+5) = f^{-1}(y)$$

This is the <u>inverse</u> of $f(x)$, in that if we give it one of the "answers" we got above, we get the original x back.

If $y = -4$, then $x = 2$

If $y = -7$, then $x = -4$

We do the same with trig functions, except we have one problem: trig functions are not 1-1 unless we ~~?~~ restrict them. Hence their inverses are not really "functions" unless restricted.

Remember: with an inverse function we are going "backward." Thus, with inverse trig functions, we give the function a <u>value</u> and get back an <u>angle</u>!

Again, if $y = \sin x$

then the inverse is written

$$y = \sin^{-1} x \quad (\text{or to avoid confusion } y = \arcsin x)$$

One gives $\underline{\sin^{-1} x}$ a value, and gets back y as an angle.

i.e. since $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

we have $\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

NOTATION

$y = \sin^{-1} x$ is the inverse function of $\underline{\sin x}$

It is also written $y = \arcsin x$ (cf some calculators)

It is NOT the reciprocal.

i.e. $y = (\sin x)^{-1} = \frac{1}{\sin x} = \csc x \neq \sin^{-1} x$