

SIGN OF FIRST DERIVATION

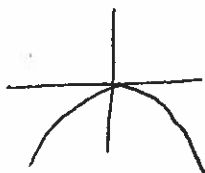
The derivative tells us the slope of a given function.

If the slope is + at a point, the function is rising (increasing)
if -, the function is falling (decreasing)

This information can be very useful when graphing functions.

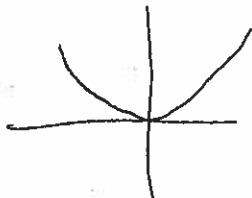
Note: The sign of the derivative is unrelated to the sign of a function itself.

EG ① Let $y = -x^2$



} $\Rightarrow$ for $x < 0$
the deriv is pos
even though y is always neg.

② Let $y = x^2$



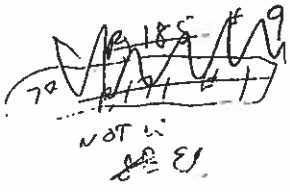
} for $x < 0$ the deriv is neg
even though y is always pos.

NOTE: If slope is 0, the curve must have a horizontal tangent.

By finding the points of zero slope, we can subdivide a curve into sub-intervals in which the curve is either rising or falling. These points (along with points where there is no slope) are called "critical pts".

V. 10

EG.



Given

$$y = x^2 - x + 1$$

what is it rising and falling?

$$\frac{dy}{dx} = 2x - 1$$

$$\text{if } \frac{dy}{dx} = 0, \text{ then } 2x - 1 = 0$$

$$\text{or } x = \frac{1}{2}$$

This is a point with horizontal tangent

Pick 2 points, one on either side of $\frac{1}{2}$, e.g. 0 and 1.

$$\left. \frac{dy}{dx} \right|_{x=0} = 2 \cdot 0 - 1 = -1 < 0$$

 $\Rightarrow$ for $x < \frac{1}{2}$ curve is falling.

$$\left. \frac{dy}{dx} \right|_{x=1} = 2 \cdot 1 - 1 = 2 - 1 = 1 > 0$$

 $\Rightarrow$ for $x > \frac{1}{2}$ curve is rising

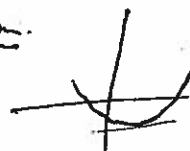
HORIZONTAL TANGENTS

{As mentioned before, if the slope^{for curve} is 0, the curve must have a horizontal tangent.

If the slope changes from pos to neg as it passes the hor. tang., at the hor. tangent it reaches a local high point or local maximum.



If the slope changes from neg to pos as it passes the hor. tang., at the hor. tang it reaches a local low point or local minimum.



It is also possible for a curve to have a horizontal tangent without either a local min or max. eg. $y = x^3$



These facts can be very useful when graphing a function.

WARNING It is possible to have a function with a min or max and no hor. tangent. eg. $y = |x|$

at $x=0$, y attains a minimum, but no derivative is defined!

CONCAVITY

7.1.2 p. 72

15 (4.4)

239 p. 239

18-4

- When a curve has a "regular-type" maximum, the slope changes from pos to zero to negative, i.e. it is decreasing.
The shape of such a curve segment is said to be concave downward (spills water).



- When a curve has a "regular-type" minimum, the slope changes from neg to zero to positive, i.e. it is increasing.
The shape of such a curve segment is said to be concave upward (holds water).



SECOND DERIVATIVE

The 1st derivative tells us the rate of change of the height (y) with respect to x .

The 2nd derivative tells us the rate of change of the 1st deriv. (slope) with respect to x .

If 2nd deriv > 0 , then the 1st deriv is increasing, i.e. the slope is increasing or we have a concave upward shape.

If 2nd deriv < 0 , then the 1st deriv is decreasing, i.e. the slope is decreasing or we have a concave downward shape.

of Thu 5
P 243
also
P 240
"2nd Deriv
Test"

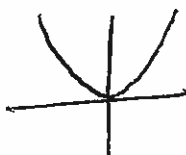
EG

①

parabola $y = x^2$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2 > 0$$

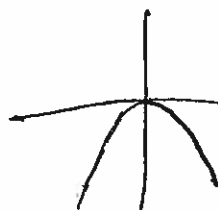


②

parabola $y = -x^2$

$$\frac{dy}{dx} = -2x$$

$$\frac{d^2y}{dx^2} = -2 < 0$$



INFLECTION



18.5
p 24

If a curve changes concavity at a point, this point is called a "point of inflection".

If 2nd deriv is pos at one point and neg at another,
I a good chance that it is 0 at some pt in between.
However, it is possible that y'' will not exist.

EXAMPLE

$$y = x^3 + 2x^2$$

$$\frac{dy}{dx} = 3x^2 + 4x$$

$$\frac{d^2y}{dx^2} = 6x + 4$$

$$\text{now if } 6x + 4 = 0$$

$$6x = -4$$

$$x = -\frac{2}{3}$$

$\therefore -\frac{2}{3}$ is a point of inflection

Test Let $x = -1$ ($< -\frac{2}{3}$)

$$\Rightarrow y''|_{x=-1} = -6 + 4 = -2 < 0 \quad \therefore \text{conc. down}$$

Let $x = 0$

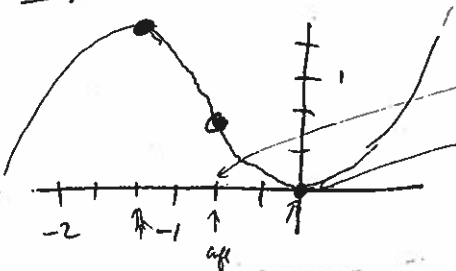
$$\Rightarrow y''|_{x=0} = 0 + 4 > 0 \quad \therefore \text{conc. up}$$

Where is there a level slope?

$$y' = 3x^2 + 4x = 0$$

$$(3x + 4)x = 0 \Rightarrow x = 0 \text{ or } x = -\frac{4}{3}$$

Graph



check heights at key points:

x	y
$-\frac{4}{3}$	$-\frac{8}{27} + \frac{6}{9} = \frac{-8}{27} + \frac{24}{27} = \frac{16}{27}$
0	0
$-\frac{4}{3}$	$-\frac{64}{27} + \frac{32}{9} = \frac{-64 + 96}{27} = \frac{32}{27}$