88-22

RELATED RATES (3.10) p 191

This is another standard application of derivatives.
In particular, it makes ~~great~~ repeated use of the chain rule.
This type of problem gets its name from 2 rates being <u>related</u> to each other. Eg.

Given $\frac{du}{dx}$ and given $y = f(u)$,
we want to find $\frac{dy}{dx}$
Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Normally 2 of the rates are time deriv.s and the other is measurement. e.g.

$$\frac{dy}{dt} = \frac{dy}{dV} \cdot \frac{dV}{dt}$$

change wrt time

REMEMBER: Derivatives tell slope — slope indicates rate (of change).
When we have problems dealing with rates, we use derivatives.

STRATEGY 14 (p 192)

1. If helpful, draw a picture
   Indicate <u>what is changing</u>.

~~2~~ 3. Indicate <u>what you want</u> (usually a rate or derivative)
   (Label it).

~~3~~ 2. Label the remaining quantities

4. Develop <u>equations linking the variables</u> together.

5. Solve, substitute, <u>differentiate</u>. (both sides w.r.t. t)

6. Evaluate at specific values given as last step.
   → more on last step.

IMPORTANT DISTINCTION

GEN FORMULA

VS

VALUE AT INSTANT

i.e. difference between 2 types of pictures.

VIDEO – includs MOTION
shows relationship of objects

PHOTO – instantaneous frozen
relationship

In many problems, we need to find change at an instant, but we must always find the general formula 1st, and, only at the end of the computations, evaluate the formula using the specific values given.

FORMULAS

Doing these problems pre-suppose one remembers various math formulas. Two often used formulas are Pythagorean thm and proportional sides of similar triangles.

EXAMPLES

There are ~~14~~ 6 examples in the Chapter pp ~~199-203~~ p 194-196

They are fairly detailed + straight-forward, so I won't go over them, but will do problems.

NOTE : you may have to review geometric formulae ~~App A 133-134~~ See back of Book

e.g. Vol of Sphere $= \frac{4}{3}\pi r^3$

Surface Area Sp $= 4\pi r^2$

14 #196

~~#3~~ #1

A = area of circle $\bar{c}$ rad r.

$\therefore \; A = \pi r^2$

How is $\frac{dA}{dt}$ related to $\frac{dr}{dt}$?

$$\Longrightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

(1.7)

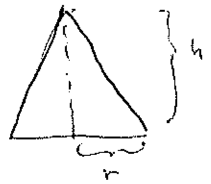
18

cf. p. 197 #27

Sand falls onto cone at rate of $10\ ft^3/min$.

We always have $r = \frac{1}{2}h$. (r = rad of base).

How fast is h increasing when cone is 5 ft deep?



We know $\underline{\text{rate of increase of } V} = 10\ ft^3/min$

i.e. $\frac{dV}{dt} = 10$

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

We want $\left.\frac{dh}{dt}\right|_{h=5}$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$$

$$\therefore \frac{dV}{dt} = \frac{3\pi h^2}{\cancel{12}_{4}}\frac{dh}{dt}$$

$$\therefore 10 = \frac{\pi h^2}{4}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{40}{\pi h^2}$$ = Gen formula

$$\left.\frac{dh}{dt}\right|_{h=5} = \frac{40}{\pi \cdot 25} = \underline{\underline{\frac{8}{5\pi}}}.$$

[illegible]

(Thomas Calculus 14^{th} ed). Pg 197, # 25

A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

pg 873 #21

NOT IN 13

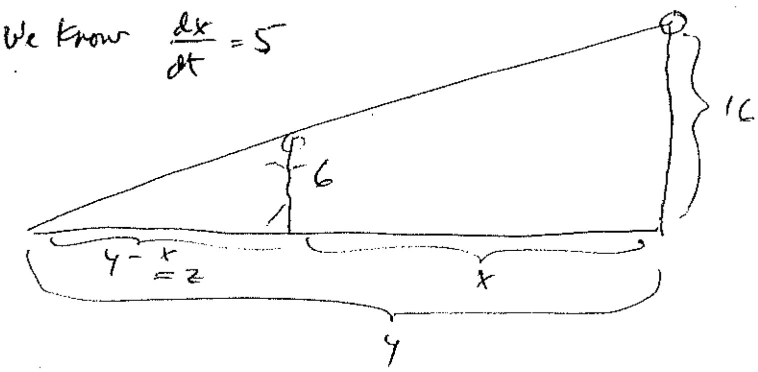
Man is 6ft tall

Walks at rate of 5ft/sec toward light 16 ft tall.

a) what is rate of tip of his shadow moving?

b) What is rate of length of shadow changing ~~at 10ft from base of light~~?

We know $\frac{dx}{dt} = 5$



a) want $\frac{dy}{dt}$.

use similar Δs, we know $\frac{y-x}{6} = \frac{y}{16}$

$\Rightarrow 16(y-x) = 6y \Rightarrow 8y - 8x = 3y \Rightarrow 5y = 8x$

$\Rightarrow y = \frac{8}{5}x$

$\therefore \frac{dy}{dt} = \frac{8}{5}\frac{dx}{dt} = \frac{8}{5}\cdot 5 = \underline{\underline{8}}$

b) Let $y - x = z$

Want $\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = 8 - 5 = \underline{\underline{3}}$