14(3.4) p 144 L.d [illegible] 92 [illegible] 86

DERIV AS RATE OF CHANGE.

# VELOCITY and RATES

92 9

To review again.
Slope is a ratio (of rise over run, or of height w.r.t. distance).

Ratio can be thought of as a rate-of-change factor.

A common rate-of-change we all use is speed or velocity — the change of distance w r t time.
Derivatives (being a limit of a ratio) can help use examine other rates-of-change.

$$\text{velocity} = \frac{\text{change in distance}}{\text{time}} \quad \text{e.g.} \quad \frac{55 \text{ miles}}{\text{hour}}$$

Using derivatives, instantaneous velocity is

14, p 145

$$v = \frac{ds}{dt} \qquad (\underline{s} \text{ is common used for } \underline{\text{distance}})$$

Similarly, acceleration is the rate of change of velocity w r t time, i.e.

p 146

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2} \quad \text{e.g.} \quad \frac{32 \text{ ft}}{\text{sec}^2} \text{ or } \frac{32 \text{ ft/sec}}{\text{sec}}$$

p 145 Speed is the abs. value of velocity.

p 146 Jerk is the deriv of accel, i.e. $j(t) = \frac{da}{dt} = \frac{d^3 s}{dt^3}$

## EXAMPLES

PS 90 #10

① Find $v = \frac{ds}{dt}$ if $s = 180t - 16t^2$.

$$v = \lim_{\Delta t \to 0} \frac{(\cancel{180t} + 180\Delta t - \cancel{16t^2} - 32t\Delta t - 16\Delta t^2) - \cancel{180t} + \cancel{16t^2}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} 180 - 32t - 16\Delta t = 180 - 32t$$

② When does $v$ vanish (i.e. $= 0$).

$$0 = 180 - 32t$$
$$180 = 32t$$
$$45 = 8t$$

$\rightarrow t = \frac{45}{8} = 5\,5/8$ sec.

We could ask also: if a stone is thrown upward and its change of location vertically is described by $s = 180t - 16t^2$, how high does it go?

① Stop + think.
at the highest point, it stops going up + starts going down
i.e. its upward velocity $= 0$
From above, when $v = 0$, $t = 5\,5/8$ sec.

② we have a formula to ~~........~~ compute vertical location,
$s = 180t - 16t^2$
plug $5\,5/8 = \frac{45}{8}$ in to this eq.

$$s = 180 \cdot \frac{45}{8} - \overset{2}{\cancel{16}} \frac{45^2}{8^2}$$

$$= \frac{180 \cdot 45 - 2 \cdot 45^2}{8} = \frac{8100 - 4050}{8} = \frac{4050}{8} = 506\,1/4 \text{ ft}$$

~~HW pg 56-58~~
~~#1, 4, 6, 10, 24, 25~~

# OTHER RATES OF CHANGE

14 Pg 144 EX 1

The area of a circle is $A = \pi r^2$ where $r$ is the radius. Let $D$ be the diameter. $\therefore\ r = \frac{D}{2}$.

$\therefore\ A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi}{4} D^2$

If we want to see how the area changes wrt the diameter we can compute $\frac{dA}{dD}$

Thus $\frac{dA}{dD} = \frac{\pi}{4} \cdot 2D = \frac{\pi D}{2}$.

If we want to find the speed when the diameter is 10 ~~feet~~ meter, we compute

$$\left.\frac{dA}{dD}\right|_{D=10} = \frac{\pi}{2} \cdot 10 = 5\pi \frac{m^2}{m} \approx 15.7 \frac{m^2}{m}$$

# MARGINAL COST

p 148

Economists often call the derivative the marginal value of a function.

If $c(x)$ is the cost to produce $x$ items in a time period, then $c'(x)$ estimates the additional cost required to produce one more item (i.e. $x+1$ items rather than $x$).

$c'(x)$ is called the marginal cost.

EX

p 149

$C(x) = x^3 - 6x^2 + 15x$ is the cost in dollars to produce $x$ radiators a day. Your shop currently produces 10 a day. How much extra to produce 11?

$$c'(x) = 3x^2 - 12x + 15$$

$$c'(10) = 3(100) - 12(10) + 15$$

$$= 195$$

The added cost is \$195 approx.

Suppose $r(x) = x^3 - 3x^2 + 12x$ gives the dollar revenue for selling $x$ radiators.

The "marginal revenue", $r'(x) = 3x^2 - 6x + 12$, estimates the increase from selling one more unit. Thus if you sell 10 radiators a day

$$r'(10) = 3(10^2) - 6 \cdot 10 + 12 = \$252$$

indicates the approx. increase in revenue if you sold 11 radiators a day.

EG 3 (ROD(N)

Pre-Note: Gravity acts s.t. if an object is dropped, it falls $16t^2$ feet after $t$ seconds.

Therefore, if we drop something from a cliff, it falls $s = 16t^2$ after $t$ seconds.

If it has some initial velocity $v_0$, this is added to the gravitational ~~velocity~~ displacement factor and we get $s = 16t^2 + v_0 t$ since the formula for distance is $s = v_0 t$.

If we change our perspective, i.e. throw something up from the ground, then $s$ is distance above the ground and we get

$$s = -16t^2 + v_0 t$$

---

Now, suppose a rock is thrown upward at an initial velocity of 144 ft/sec, then the distance formula is

$$s = 144t - 16t^2$$

$$\text{velocity} = \frac{ds}{dt} = 144 - 32t \quad [\text{ft/sec}]$$

$$\text{acceleration} = \frac{d^2s}{dt^2} = -32 \quad [\text{ft/sec}^2]$$

(a) When does it hit the ground?

Think – it hits the ground when $s = 0$.

$$0 = 144t - 16t^2 = (9 - t)16t$$

$$\underline{\underline{t = 9}} \qquad \cancel{t = 0}$$

after 9 sec.

(b) How high does it go?

at highest point, velocity $= 0$ – it stops going up + starts going down.

$$0 = 144 - 32t \Rightarrow t = \frac{144}{32} = \frac{9}{2} = 4.5 \text{ sec.}$$

But we want height! Put into distance formula.

$$s = 144\left(\tfrac{9}{2}\right) - 16\left(\tfrac{9}{2}\right)^2 = 72 \cdot 9 - 4 \cdot 81 = 648 - 324 = 324 \,//$$

max height

(c) How fast is it going when it reaches 224 ft?

Speed is based on $t$. We first need to find $t$ when it reaches 224 ft.

$$224\,ft = 144t - 16t^2$$
$$56 = 36t - 4t^2$$
$$14 = 9t - t^2$$
$$t^2 - 9t + 14 = 0$$
$$(t-7)(t-2) = 0 \qquad t = 7, 2$$

going up, $t=2$, going down $t=7$.

$t=2$

$$v = 144 - 32t = 144 - 32(2) = 144 - 64 = 80\ ft/sec$$

$t=7$

$$v = 144 - 32t = 144 - 32(7) = 144 - 224 = -80\ ft/sec$$

i.e. 80 ft/sec <u>down</u>.