A. INTRO

1st week – review of algebra + trig. – most of Chpt 1.

May be boring, but provides foundation for Math 11 + 12

Some important topics are also in App. A.1 and A.3
esp. eqs. of lines.

Cannot review everything. Sometimes you may have
to consult portions of the book.

Next week, Chpt 2, look at more "calculus"-related
topics.

EXAMPLES

B 2 EX 1

1) $y = x^2$

Domain of x → no restrictions → $-\infty < x < \infty$ or $x \in (-\infty, \infty)$

Range of y → cannot be neg. → $0 \le y$ or $y \in [0, \infty)$

2) $y = \sqrt{1 - x^2}$

Domain of x → cannot have a value which make $1 - x^2$ negative

$\therefore$ x^2 must be ≤ 1

$\therefore$ $-1 \le x \le 1$ or $x \in [-1, 1]$

Range of y → Note: if $x = \pm 1$, $y = 0$

if $x = 0$, $y = 1$

y can't be < 0 or > 1

$\therefore$ $0 \le y \le 1$ or $y \in [0, 1]$

NOTE functions can be expressed

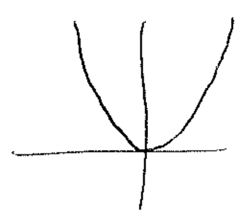
cf. p 3 EX 2

a) algebraically $y = x^2$

b) via tables (discrete values)

x	y
-2	4
-1	1
0	0
1	1
2	4

c) graphically

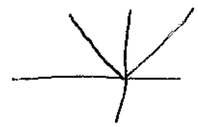


SPECIAL FUNCTIONS

PS5

1. PIECEWISE (CONTINUOUS)

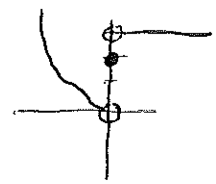
eg. a) $y = |x|$ absolute value



cf. AP-4-5

or $y = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x > 0 \end{cases}$

b) $y = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & x = 0 \\ 3 & x > 0 \end{cases}$



2. DISCONNECTED / DISCONTINUOUS

eg. $y = \lfloor x \rfloor$ greatest integer or "floor" function

$\lfloor \pi \rfloor = 3$ $\quad \lfloor e \rfloor = 2$ $\quad$ = greatest integer less than argument.

$\lfloor -1.5 \rfloor = -2$

SPECIAL QUALITIES

1. INCREASING/DECREASING

<u>Def</u> If f is a function defined on an interval and x_1, x_2 are in the interval s.t. $x_1 < x_2$,

Then

a) if $f(x_2) > f(x_1)$ whenever $x_1 < x_2$ then f is increasing

b) if $f(x_2) < f(x_1)$ whenever $x_1 < x_2$ then f is decreasing.

Eg. $y = x^3$ is an increasing function on the real line

$y = -x$ is a decreasing function on the real line.

2. EVEN/ODD (SYMMETRY ABOUT y-axis OR ORIGIN)

<u>Def</u> A function f is an

a) EVEN function of x if $f(-x) = f(x)$

b) ODD function of x if $f(-x) = -f(x)$

Eg. $y = x^2$ is an <u>even</u> function.

$y = x^3$ is an <u>odd</u> function

<u>NOTE:</u> EVEN functions are symmetry about the <u>y-axis</u>. (MIRROR symmetry)

ODD functions are symmetry about the origin. (TV-symmetry)

— or — one can turn graph upside down and the graph is the same

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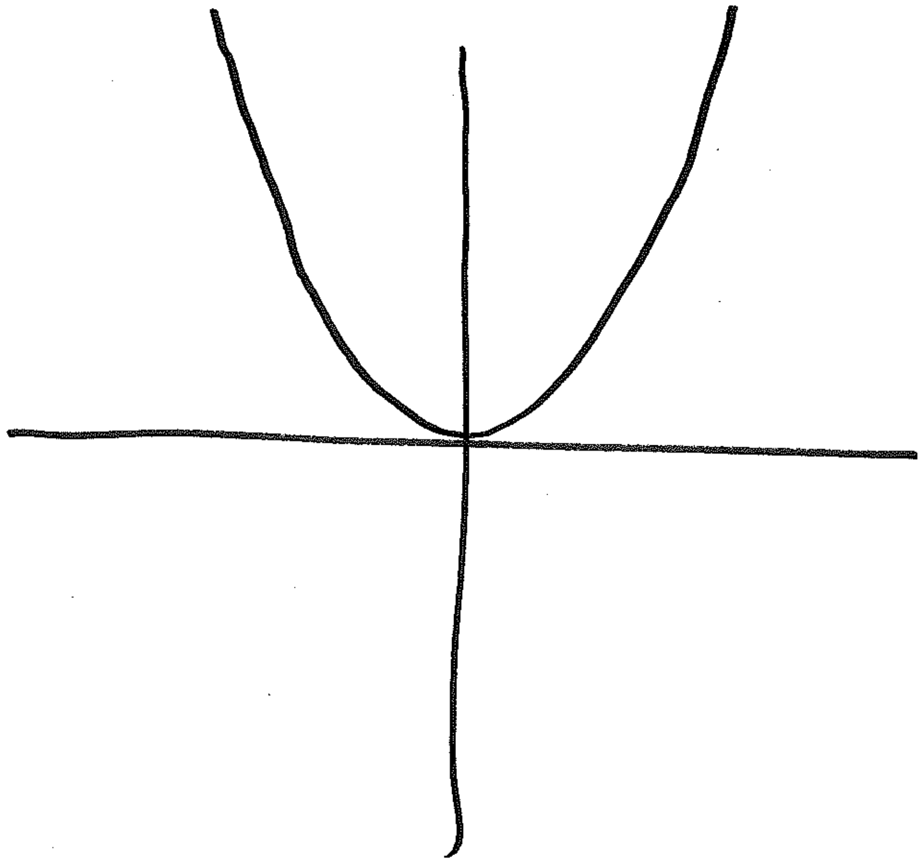
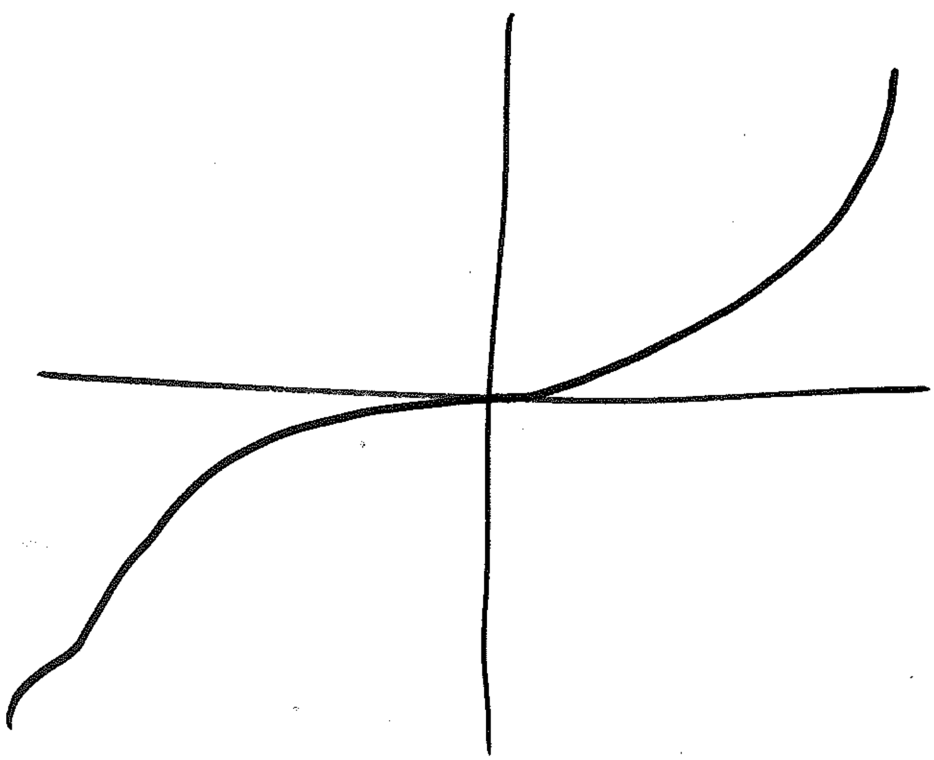
NOTE: If one negates $f(x)$ to get $-f(x)$, one produces a "reflection" of $f(x)$ across the <u>x-axis</u>

p17





COMBINING FUNCTIONS (1, 2)

p. 14 1) ARITHMETIC OPERATIONS

Given $f(x)$, $g(x)$ and domains D_f, D_g, we can combine the functions using the 4 arithmetic operations, eg. $f(x)+g(x)$. However, the new function is defined only where the two original domains overlap.

Eg. if $f(x)=\sqrt{x}$ $\quad D_f=[0,\infty)$

if $g(x)=\sqrt{1-x}$ $\quad D_g=(-\infty,1]$

Thus $f(x)-g(x)=\sqrt{x}-\sqrt{1-x}$ is defined on $[0,1]$ at most.

p15 2) "COMPOSITION" OF FUNCTIONS

Joining f and g together s.t. the output of g becomes the input to f is called the composition of f with g, written

$$f\circ g(x)=f(g(x))$$

NOTE: This is not the same as $f(x)\cdot g(x)$.

Eg. If $f(x)=x-7$ and $g(x)=x^2$, let's compute $f(g(2))$.

$g(2)=2^2=4$ $\quad f(4)=4-7=-3$

NOTE: $g(f(2))$

$f(2)=2-7=-5$ $\quad g(-5)=25$

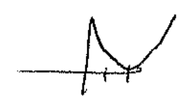
$$\therefore\ f\circ g(x)\neq g\circ f(x)$$

SHIFTING (GRAPHS OF) FUNCTIONS (p 16)

Given $y = x^2$ 

$y = x^2 + 1$ or $y - 1 = x^2$ shifts the function one unit up the y-axis 

In general, $y - k = f(x)$ is the function $y = f(x)$ shifted $\underline{k}$ units on y-axis.

Given $y = x^2$, $y = (x-2)^2$ shifts the function two units along the x-axis in the positive direction 

In general, $y = f(x - h)$ is the function $y = f(x)$ shifted $\underline{h}$ units on the x-axis.

SCALING FUNCTIONS (p 17)

By multiplying $f(x)$ by an appropriate constant, one stretches/compresses the vertical aspect of f.

By multiplying x by an appropriate constant, one stretches/compresses the horizontal aspect of f.