perfect scores per problem

Math 11-3 (8:00 AM)--Fall, 2023
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Midterm III (45 minutes)
Friday, November 3, 2023

19/30 1. (12) Find $\frac{dy}{dx}$ given $xy^2 + x^2y = 6$.

15/30 2. (14) Given $f(x) = x^3 - 1$, (a) find the inverse function, and (b) show this new function is, in fact, the inverse by evaluating $f(2)$ and then evaluating the new function at $f(2)$.

15/30 3. (12) Find $\frac{dy}{dx}$ given $y = \ln([\ln(x^2)]\sin x)$.

20/30 4. (12) Find $\frac{dy}{dx}$ given $y = x\arcsin(x^2)$.
EASIEST - TIE

18/30 5. (12) Find $\frac{dy}{dx}$ given $y = \arctan(3\sin(x^2))$.

20/30 6. (12) Given that $y = \arcsin(x^2)$, find the corresponding value of $\cot y$ (without any inverse trig functions in the answer).
EASIEST - TIE

17/30 7. (14) Find the equation of the line tangent to the curve $y = \ln(x^2)$ at $x = e$.

8/30 8. (12) The cross section of an artery can be imagined to be a circle. Suppose the radius of a given artery is 1.5 cm and that fatty deposits are being uniformly built up inside the artery at the rate of 0.2 cm per month. When the deposit is 0.4 cm thick, at what rate is the cross-sectional area (which is still open) of the artery changing?
HARDEST

REMEMBER!

$\Longrightarrow$ Please start EACH problem on a NEW PAGE SIDE!
$\Longrightarrow$ PLEASE *label* (or underline or box in) all ANSWERS clearly.
$\Longrightarrow$ Remember to show all work for full credit.
$\Longrightarrow$ ALSO remember, *NO CALCULATORS!*

Thanks.

STATS MED <84> σ 15.11
Hi 100 MEAN 82.43 # EXAMS 30
LO 41

Mt 11 – MID III – F23

1. $xy^2+x^2y=6 \Rightarrow x\,2y\frac{dy}{dx}+y^2\cdot 1+x^2\frac{dy}{dx}+y\cdot 2x=0$

$\Rightarrow (2xy+x^2)\frac{dy}{dx}=-(y^2+2xy) \Rightarrow \frac{dy}{dx}=-\frac{y^2+2xy}{x^2+2xy}$

2. $f(x)=x^3-1$

a) $y=x^3-1 \Rightarrow y+1=x^3 \Rightarrow x=\sqrt[3]{y+1}$

b) $f(2)=2^3-1=8-1=7$

$x=\sqrt[3]{7+1}=\sqrt[3]{8}=2$

3. $y=\ln\left((\ln x^2)\sin x\right) \Rightarrow \frac{dy}{dx}=\frac{1}{(\ln x^2)(\sin x)}\cdot\left[(\ln x^2)\cos x+(\sin x)\frac{1}{x^2}\cdot 2x\right]$

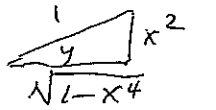
$=\frac{1}{(\ln x^2)(\sin x)}\left[(\cos x)\ln x^2+\frac{2\sin x}{x}\right]$

4. $y=x\arcsin x^2 \Rightarrow \frac{dy}{dx}=x\frac{1}{\sqrt{1-x^4}}\cdot 2x+(\arcsin x^2)\cdot 1$

$=\frac{2x^2}{\sqrt{1-x^4}}+\arcsin x^2$

5. $y=\arctan\left(3\sin(x^2)\right) \Rightarrow \frac{dy}{dx}=\frac{1}{1+9\sin^2(x^2)}\cdot(3\cos x^2)2x$

$=\frac{6x\cos x^2}{1+9\sin^2(x^2)}$

6. $y=\arcsin x^2 \Rightarrow x^2=\sin y \Rightarrow$ 

$\Rightarrow \cot y=\frac{\sqrt{1-x^4}}{x^2}$

7. $y=\ln(x^2) \Rightarrow \frac{dy}{dx}=\frac{1}{x^2}\cdot 2x=\frac{2}{x}\Big|_{x=e}=\frac{2}{e}$ $\qquad y\Big|_{x=e}=\ln e^2=2\ln e=2$

$\therefore$ tangent line: $\frac{y-2}{x-e}=\frac{2}{e} \Rightarrow y-2=\frac{2}{e}(x-e)=\frac{2}{e}x-2 \Rightarrow y=\frac{2}{e}x$

8. 

$r_0=1.5$

$\frac{dh}{dt}=.2\ \frac{\text{cm}}{\text{month}}$

$r=r_0-h=1.5-h$

want $\left.\frac{dA}{dt}\right|_{h=.4}$ $\qquad A=\pi r^2=\pi(1.5-h)^2=\pi(2.25-3h+h^2)$

$\frac{dA}{dt}=\pi(-3+2h)\frac{dh}{dt}$

$\left.\frac{dA}{dt}\right|_{h=.4}=\pi(-3+2(.4))(.2)=\pi(-3+.8)(.2)$

$=\pi(-2.2)(.2)=-.44\pi$