perfect scores per problem

Math 11-3 (8:00 AM)—Fall, 2023
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Midterm II (45 minutes)
Friday, October 20, 2023

14/33 1. (30) Find $\frac{dy}{dx}$ for each of the following.

(a) $y = (5x^4 - 3x^2)(2x - 5)^3$ (b) $y = 62x^2 - 51.3x - \sqrt{10}$

(c) $y = \frac{x+1}{(x^2-3x)^{1/2}}$ (d) $y = 5\pi^2$

(e) $y = e^x \tan x$

22/33 2. (12) Generalization lies at the very center of mathematics. That is, given a specific result, the mathematician wants to generalize to more cases. Using rules you already know, derive a "nice" formula (i.e., a formula WITHOUT any parentheses in which *only* u, v, w, u', v', and w' appear) for the following. (Note that this is merely a generalization for the derivative of a product of two functions of x.) *Show all steps of the derivation for full credit!*

$$\frac{d}{dx}(uvw)$$

11/33 HARDEST 3. (10) Evaluate: $\lim_{\theta \to 0} \frac{\tan 5\theta}{3\theta}$.

20/33 4. (12) Let $y = (2x + 4)^3$ be a curve in the plane. Find the equations of the lines that are tangent and normal to this curve at $(-1, 8)$. (Remember that a *normal* line is perpendicular to the *tangent* line.)

28/33 EASIEST 5. (12) A baseball is thrown up into the air and the altitude in feet is described by $s(t) = 64t - 16t^2$. Using this formula and the associated formula for the velocity:

(a) determine how long it takes the baseball to reach its highest point,

(b) determine the height at the highest point, and

(c) determine how fast the ball is going after 1 second.

18/33 6. (12) Find $\frac{dy}{dx}$ given $y = \cos^3(5\cos 4x^2)$.

20/33 7. (12) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ given $y = 4x^3 \cos 2x$.

REMEMBER! Please start EACH problem on a NEW PAGE SIDE!
ALSO, please PLEASE *label* (or underline or box in) all ANSWERS clearly.
Thanks.

STATS

HI 100 MEDIAN 91 σ 9.40
LO 13 MEAN 88.45 # EXAMS 33
(ADJ)

M+ 11 – F23 – M102

① a) $y = (5x^4 - 3x^2)(2x-5)^3 \Rightarrow \frac{dy}{dx} = (5x^4-3x^2)3(2x-5)^2 \cdot 2 + (2x-5)^3(20x^3-6x)$

$[= 10x(2x-5)^2(7x^3-10x^2-3x+3)]$

b) $y = 62x^2 - 51.3x - \sqrt{10} \Rightarrow \frac{dy}{dx} = 124x - 51.3$

c) $y = \frac{x+1}{(x^2-3x)^{1/2}} \Rightarrow \frac{dy}{dx} = \frac{(x^2-3x)^{1/2}\cdot 1 - (x+1)\left(\frac{1}{2}\right)(x^2-3x)^{-1/2}(2x-3)}{x^2-3x} = \left[\frac{-5x+3}{2(x^2-3x)^{3/2}}\right]$

d) $y = 5\pi^2 \Rightarrow \frac{dy}{dx} = 0$

e) $y = e^x \tan x \Rightarrow \frac{dy}{dx} = e^x \sec^2 x + e^x \tan x$

② $\frac{d}{dx}(uvw) = \frac{d}{dx}(u(vw)) = u\frac{d}{dx}(vw) + u'vw = u(vw' + v'w) + u'vw$

$= uvw' + uv'w + u'vw$

③ $\lim_{\theta\to 0} \frac{\tan 5\theta}{3\theta} = \lim \frac{\sin 5\theta}{5\theta} \cdot \frac{5}{3\cos 5\theta} = \frac{5}{3}$ ($\frac{\sin 5\theta}{5\theta} \to 1$, $\cos 5\theta \to 1$)

④ $y = (2x+4)^3 \Rightarrow \frac{dy}{dx} = 3(2x+4)^2 \cdot 2 = 6(2x+4)^2$ at $x=-1$, $\frac{dy}{dx} = 6(-2+4)^2 = 6\cdot 4 = 24$

tangent line $24 = \frac{y-8}{x+1}$ $[y = 24x + 32]$ normal line $\frac{-1}{24} = \frac{y-8}{x+1}$ $\left[y = \frac{-x}{24} + \frac{191}{24}\right]$

⑤ $s(t) = 64t - 16t^2 \Rightarrow v(t) = 64 - 32t$

a) highest pt is when $v=0 \Rightarrow 0 = 64 - 32t \Rightarrow 64 = 32t \Rightarrow t = 2$

b) $s(2) = 64\cdot 2 - 16\cdot 2^2 = 128 - 64 = 64$

c) $v(1) = 64 - 32\cdot 1 = 32$

⑥ $y = \cos^3(5\cos 4x^2) \Rightarrow \frac{dy}{dx} = 3\cos^2(5\cos 4x^2)(-\sin(5\cos 4x^2))(-5\sin 4x^2)\,8x$

⑦ $y = 4x^3 \cos 2x \Rightarrow \frac{dy}{dx} = -4x^3(\sin 2x)2 + (\cos 2x)12x^2$

$= -8x^3 \sin 2x + 12x^2 \cos 2x$

$\Rightarrow \frac{d^2y}{dx^2} = -8x^3(\cos 2x)2 + (\sin 2x)(-24x^2) + 12x^2(-\sin 2x)2 + (\cos 2x)(24x)$

$= -16x^3\cos 2x - 48x^2 \sin 2x + 24x\cos 2x$