Math 11-3 (8:00 AM)—Fall, 2023
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Midterm I (45 minutes)
Friday, Oct 6, 2023

NOTES:

1. Make sure your NAME is on the front of the green book.
2. PLEASE *label* (or underline or $\boxed{\text{box in}}$) all ANSWERS clearly.
3. Start EACH problem on a NEW PAGE SIDE!
4. Problems can be written in the blue book in ANY ORDER, but they must be properly LABELED.
5. There are 100 points possible on this test. The point value of each problem is listed in parenthesis after the number.
6. *Show your WORK!* — partial credit is possible only when all work needed to obtain an answer is presented legibly.
7. **NO CALCULATORS.**

perfect scores per problem

10/33 1. (12) If $f(x) = \dfrac{x}{x+1}$ and $g(x) = x - \dfrac{1}{x}$, find (a) $f(g(x))$ and (b) $g(f(x))$. The final answers should NOT have any fractions in the numerators or denominators nor be a sum or difference of two separate fractions.

29/33 2. (8) Evaluate: $\displaystyle\lim_{x\to 1} \frac{(x^2-4)(x-1)}{x^2-3x+2}$

23/33 3. (8) Evaluate: $\displaystyle\lim_{t\to 3} \frac{2t^3-18t}{3t-9}$

30/33 EASIEST 4. (12) If $f(x) = \begin{cases} x^2-1 & \text{for } x<1 \\ \lfloor x+1 \rfloor & \text{for } x \ge 1 \end{cases}$

(a) what is $\displaystyle\lim_{x\to 1^+} f(x)$? (b) what is $\displaystyle\lim_{x\to 1^-} f(x)$?

21/33 5. (15) Using ONLY the *definition* of the derivative (and not anything you learned from other courses!), calculate the derivative of $f(x) = 2x^2 - 2x$.

12/33 6. (14) Let $f(x) = \dfrac{(x-5)(x+2)(x-1)}{x^2+x-2}$.

(a) What are the point(s) of discontinuity?

(b) How should $f(x)$ be defined at this (these) point(s) to make it continuous (i.e., can you "remove" this (these) point(s) to make a "continuous extension")?

7/33 HARDEST 7. (16) Evaluate:

(a) $\displaystyle\lim_{y\to 0} \frac{\sin 2y}{2\cos y}$. (b) $\displaystyle\lim_{\theta\to 0} \frac{\sin 3\theta}{4\theta}$.

25/33 8. (15) What is (are) the asymptote(s) of $y = \dfrac{x^2+1}{x+1}$? For each asymptote, give its type (i.e., *vertical, horizontal,* or *oblique*).

STATS

HI 100 MEDIAN 84 $\sigma = 18.204$

LO 27 MEAN 79.27 # EXAMS 33

M+11 − F23 − MID I

1. $f(x)=\frac{x}{x+1}$ $g(x)=x-\frac{1}{x}$ a) $f(g(x))=\dfrac{x-\frac{1}{x}}{x-\frac{1}{x}+1}=\dfrac{\frac{x^2-1}{x}}{\frac{x^2-1+x}{x}}=\dfrac{x^2-1}{x^2+x-1}$.

b) $g(f(x))=\dfrac{x}{x+1}-\dfrac{1}{\frac{x}{x+1}}=\dfrac{x}{x+1}-\dfrac{x+1}{x}=\dfrac{x^2-(x+1)^2}{x(x+1)}=\dfrac{\cancel{x^2}-\cancel{x^2}-2x-1}{x^2+x}=\dfrac{-2x-1}{x^2+x}$

2. $\lim\limits_{x\to 1}\dfrac{(x^2-4)(x-1)}{x^2-3x+2}=\lim\dfrac{(x+2)\cancel{(x-2)}\cancel{(x-1)}}{\cancel{(x-2)}\cancel{(x-1)}}=\lim x+2=1+2=\underline{3}$

3. $\lim\limits_{t\to 3}\dfrac{2t^3-18t}{3t-9}=\lim\dfrac{2t(t^2-9)}{3(t-3)}=\lim\dfrac{2t\cancel{(t-3)}(t+3)}{3\cancel{(t-3)}}=\dfrac{2\cdot\cancel{3}\cdot 6}{\cancel{3}}=\underline{12}$

4. $f(x)=\begin{cases}x^2-1 & x<1\\ \lfloor x+1\rfloor & x\geq 1\end{cases}$

a) $\lim\limits_{x\to 1^+}f(x)=\lim\lfloor x+1\rfloor=\lfloor 1+1\rfloor=\underline{\underline{2}}$ b) $\lim\limits_{x\to 1^-}f(x)=\lim x^2-1=1-1=\underline{\underline{0}}$

5. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\dfrac{2(x+h)^2-2(x+h)-(2x^2-2x)}{h}=\lim\dfrac{\cancel{2x^2}+4xh+2h^2-\cancel{2x}-2h-\cancel{2x^2}+\cancel{2x}}{h}$

$=\lim\dfrac{4x\cancel{h}+2h^{\cancel{2}}-2\cancel{h}}{\cancel{h}}=\lim 4x+2\cancel{h}-2=\underline{4x-2}$

6. $f(x)=\dfrac{(x-5)(x+2)(x-1)}{x^2+x-2}=\dfrac{(x-5)(x+2)(x-1)}{(x+2)(x-1)}\Rightarrow$ a) $\underline{x=1,\ x=-2\ \text{pts of discont.}}$

b) $\lim\limits_{x\to 1}f(x)=\lim(x-5)=1-5=-4$ $\quad\underline{f(1)=-4}$

$\lim\limits_{x\to -2}f(x)=\lim(x-5)=-2-5=-7$ $\quad\underline{f(-2)=-7}$

7. a) $\lim\limits_{y\to 0}\dfrac{\sin 2y}{2\cos y}=\lim\dfrac{\cancel{2}\sin y\cancel{\cos y}}{\cancel{2}\cancel{\cos y}}=\lim\sin y=\underline{0}$

b) $\lim\limits_{\theta\to 0}\dfrac{\sin 3\theta}{4\theta}=\lim\dfrac{\sin 3\theta}{3\theta}\cdot\dfrac{3}{4}=\dfrac{3}{4}$

8. $y=\dfrac{x^2+1}{x+1}\Rightarrow$

$$\begin{array}{r|l} & x-1\\ \hline x+1 & x^2+0x+1\\ & x^2+x\\ \hline & -x\\ & -x-1\\ \hline & 2\end{array}$$

$y=x-1+\dfrac{2}{x+1}$

vert asympt at $x=-1$

oblique asympt. $\underline{y=x-1}$