Math 11-3 (8:00 AM)—Fall, 2023
D.C. Smolarski, S.J.

Final Exam (2 hours maximum)
Monday, December 4, 2023 (9:10 AM)

perfect scores per problem

REMEMBER!

PLEASE start each problem at the TOP OF A NEW PAGE or the TOP OF NEW SIDE OF A PAGE!

ALSO, please *label* (or <u>underline</u> or $\boxed{\text{box in}}$) your answer. And **NO CALCULATORS!** Thanks.

NOTE: *THERE ARE 12 PROBLEMS AND TWO SIDES TO THIS FINAL EXAM SHEET! TURN THE SHEET OVER FOR THE OTHER PROBLEMS!!!!*

1. (10) (*Taken from Midterm I*) Evaluate:

19/31

(a) $\lim_{y\to 0} \dfrac{\sin 2y}{2\cos y}$. (b) $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{4\theta}$.

EASIEST-TIE

19/31

2. (10) (*Taken from Midterm II*) Find $\dfrac{dy}{dx}$ given $y = \cos^3(5\cos 4x^2)$.

18/31

3. (10) (*Taken from Midterm III and the sample final* and the sample final) The cross section of an artery can be imagined to be a circle. Suppose the radius of a given artery is 1.5 cm and that fatty deposits are being uniformly built up inside the artery at the rate of 0.2 cm per month. When the deposit is 0.4 cm thick, at what rate is the cross-sectional area (which is still open) of the artery changing?

18/31

4. (10) (*Taken from sample final*) A metal can is to be made to hold 1 liter of oil. When making the cylindrical side of the can, there is no wasted metal, but the top and bottom circular pieces must be cut from squares of material and the excess is wasted. Thus, the cost of material must take into account that the end pieces of the can are made from squares, rather than from circles. On the other hand, the volume is computed assuming the top and bottom are circles. Find the dimensions (i.e., of the *radius* and the *height*) of the can that will minimize the cost of the metal to manufacture it.

8/31

5. (10) Evaluate the following limits:

(a) $\lim_{x\to\infty} \sqrt{x^2+3x+1} - \sqrt{x^2+1}$ (b) $\lim_{t\to\infty} \dfrac{t^2}{3t+1}$

16/31

6. (10) A rancher wants to enclose two rectangular pens (both equal in area) bordering on a river, one for sheep and one for cattle. He has 240 feet of fencing to use, and (to save on fencing) one section of fence will separate the two pens and the side of each pen facing the river will not be fenced. What is the MAXIMUM TOTAL AREA he can enclose?

16/31

7. (9) Find the asymptote(s) of $y = \dfrac{x^2-4}{x-1}$ and indicate which type (i.e., horizontal, vertical, or oblique) each is.

(⟹ *Remainder of exam on other side!!!)*

2/31 8. (12) A weight is attached to a rope 50 feet long which passes over a pulley fastened 20 feet above the ground. The other end of the rope is attached to the rear bumper of a truck, 2 feet above the ground. If the truck moves off at the rate of 9 feet per second, how fast is the weight rising when it is 6 feet above the ground?

18/31 9. (10) A rock thrown vertically upward from the surface of the moon at an initial velocity of 24 meters per second reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.

(a) How high does the rock go?

(b) How long does it take the rock to reach its highest point?

10. (16) Find $\dfrac{dy}{dx}$

8/31

(a) $y = (\sin x)^{\cos x}$

(b) $y = (x+y)^3$

(c) $y = \arcsin(3x^2)$

(d) $y = \ln(\ln x^2)$

13/31 11. (9) Find the linearization $L(x)$ of $f(x) = \sqrt{x^2+9}$ at $x = -4$.

18/31 12. (9) Given the general Newton's Method formula of $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, given $x_0 = 2$ and $f(x) - 2x - x^2 + 1$, find x_2.

125 points total

P.S. Have a restful Christmas break!

STATS

HI 125/125 MEDIAN 93 σ 16.84

LO 38 MEAN 96.13 # EXAMS 31

MH 11 – F 23 – FINAL

1 a) $\lim_{y\to 0} \frac{\sin 2y}{2\cos y} = \lim \frac{\cancel{2}\sin y\,\cancel{\cos y}}{\cancel{2\cos y}} = \lim \sin y = \underline{0}$

b) $\lim_{\theta\to 0} \frac{\sin 3\theta}{4\theta} = \lim \cancel{\frac{\sin 3\theta}{3\theta}}^{\,1} \cdot \frac{3}{4} = \frac{3}{4}$

2. $y = \cos^3(5\cos 4x^2) \Rightarrow \frac{dy}{dx} = 3\cos^2(5\cos 4x^2)\cdot\left(-\sin(5\cos 4x^2)\right)(-5\sin 4x^2)(8x)$

3. 

$r_0 = 1.5$

$\frac{dh}{dt} = .2$ cm/month

want $\left.\frac{dA}{dt}\right|_{h=.4}$

$A = \pi r^2 = \pi(1.5-h)^2 = \pi(2.25 - 3h + h^2)$

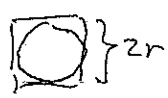
$\Rightarrow \frac{dA}{dt} = \pi(-3+2h)\frac{dh}{dt}$

$r = r_0 - h = 1.5 - h$

$\Rightarrow \left.\frac{dA}{dt}\right|_{h=.4} = \pi(-3+2(.4))(.2) = \pi(-3+.8)(.2)$

$= \pi(-2.2)(.2) = \underline{-.44\pi}$

4. 



$V = 1 \text{ liter} = \pi r^2 h \longrightarrow h = \frac{1}{\pi r^2}$

$A = \underbrace{2\pi r h}_{\text{side}} + \underbrace{2(2r)^2}_{\text{top+bottom}}$

$= 2\cancel{\pi}\cancel{r}\left(\frac{1}{\cancel{\pi} r^{\cancel{2}}}\right) + 8r^2 = 2r^{-1} + 8r^2$

$\frac{dA}{dr} = 2(-1)r^{-2} + 16r = \frac{-2}{r^2} + 16r = \frac{16r^3 - 2}{r^2}$

$\frac{dA}{dr} = 0 \Rightarrow 16r^3 - 2 = 0 \Rightarrow r^3 = \frac{2}{16} = \frac{1}{8} \Rightarrow \underline{r = \frac{1}{2}}$

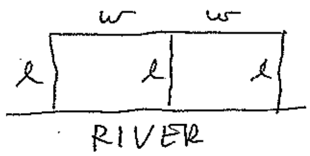
$h = \frac{1}{\pi\left(\frac{1}{2}\right)^2} = \underline{\frac{4}{\pi}}$

5. a) $\lim_{x\to\infty} \sqrt{x^2+3x+1} - \sqrt{x^2+1} = \lim \left(\sqrt{x^2+3x+1} - \sqrt{x^2+1}\right)\left(\frac{\sqrt{x^2+3x+1} + \sqrt{x^2+1}}{\sqrt{x^2+3x+1} + \sqrt{x^2+1}}\right)$

$= \lim \frac{\cancel{x^2} + 3x + \cancel{1} - (\cancel{x^2} + \cancel{1})}{\sqrt{x^2+3x+1} + \sqrt{x^2+1}} = \lim \frac{3x}{\sqrt{x^2+3x+1} + \sqrt{x^2+1}}\;\frac{1/x}{1/x} = \lim \frac{3}{\sqrt{1 + \underbrace{\cancel{3/x}}_{\to 0} + \underbrace{\cancel{1/x^2}}_{\to 0}} + \sqrt{1 + \underbrace{1/x^2}_{\to 0}}}$

$= \frac{3}{1+1} = \underline{\frac{3}{2}}$

b) $\lim_{t\to\infty} \frac{t^2}{3t+1} = \lim \frac{t^2}{3t+1}\;\frac{1/t^2}{1/t^2} = \lim \frac{1}{\underbrace{\cancel{\frac{3}{t}}}_{\to 0} + \cancel{\frac{1}{t^2}}} = \underline{\infty}$

6 

$\Rightarrow \quad 3l + 2w = 240$
$\quad 2w = 240 - 3l$
$\quad w = 120 - \frac{3}{2}l$

$A = 2w \cdot l$
$= 2(120 - \frac{3}{2}l)l$
$= 240l - 3l^2$

$\frac{dA}{dl} = 240 - 6l = 0$
$\Rightarrow 6l = 240$
$\Rightarrow l = 40$

$\text{Area} = 240 \cdot 40 - 3 \cdot 40^2$
$= 9600 - 3(1600)$
$= 9600 - 4800 = \underline{4800}$

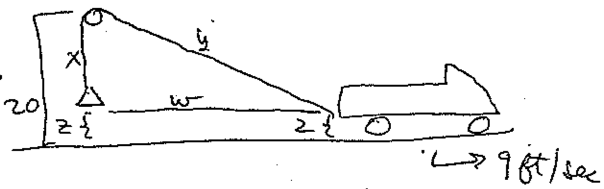
7. $y = \frac{x^2 - 4}{x - 1}$

$$\begin{array}{r} x + 1 \\ x-1 \overline{) x^2 + 0x - 4} \\ \underline{x^2 - x} \\ x - 4 \\ \underline{x - 1} \\ -3 \end{array}$$

$\Rightarrow y = x + 1 - \frac{3}{x-1}$

$\Rightarrow \underline{y = x + 1}$ $\underline{\text{oblique}}$ asymptote

$\underline{x = 1}$ $\underline{\text{vertical}}$ asymptote

8. 

$\begin{cases} x + z = 20 \\ x + y = 50 \end{cases}$

$\frac{dw}{dt} = 9$

$y = 50 - x$
$= 50 - (20 - z)$
$= 30 + z$

want $\left.\frac{dz}{dt}\right|_{z=6}$

$$\begin{array}{r} 36 \\ 36 \\ \hline 216 \\ 108 \\ \hline 1296 \end{array} \qquad \begin{array}{r} 18 \\ 18 \\ \hline 144 \\ 18 \\ \hline 324 \end{array}$$

at $z = 6$
$w^2 = 36^2 - 18^2$
$= 1296 - 324 = 972$
$= 81 \cdot 4 \cdot 3$
$w = 9 \cdot 2 \cdot \sqrt{3} = 18\sqrt{3}$

$w^2 + 18^2 = y^2 = (30 + z)^2$

$\Rightarrow \cancel{2} w \frac{dw}{dt} = \cancel{2}(30 + z)\frac{dz}{dt}$

$\Rightarrow \frac{dz}{dt} = \frac{w}{30 + z} \left.\frac{dw}{dt}\right|_{z=6} = \frac{18\sqrt{3}}{30 + 6} \cdot 9 = \frac{\cancel{18}}{\cancel{36}_2} 9\sqrt{3}$

$= \underline{\frac{9\sqrt{3}}{2}}$

9. $s = 24t - 0.8t^2$

$\Rightarrow v = 24 - 1.6t$

a) highest pt $\Rightarrow v = 0 \Rightarrow 1.6t - 24 \Rightarrow t = \frac{24}{1.6} = \frac{3}{.2} = 15.$

$\Rightarrow s = 24 \cdot 15 - .8(15)^2 = 360 - 180 = \underline{180}$

$$\begin{array}{r} 24 \\ 15 \\ \hline 120 \\ 24 \\ \hline 360 \end{array} \qquad \begin{array}{r} 225 \\ .8 \\ \hline 180.0 \end{array}$$

b) $t = 15.$

10. a) $y = (\sin x)^{\cos x} \Rightarrow \ln y = \ln\left[(\sin x)^{\cos x}\right] = (\cos x)(\ln(\sin x))$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (-\sin x)\ln(\sin x) + \cos x \frac{1}{\sin x} \cdot \cos x$

$\Rightarrow \frac{dy}{dx} = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - (\sin x)\ln(\sin x)\right]$

b) $y = (x+y)^3 \Rightarrow \frac{dy}{dx} = 3(x+y)^2\left(1 + \frac{dy}{dx}\right) = 3(x+y)^2 + 3(x+y)^2\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} - 3(x+y)^2\frac{dy}{dx} = 3(x+y)^2 \Rightarrow \frac{dy}{dx} = \frac{3(x+y)^2}{1-3(x+y)^2}$

c) $y = \arcsin(3x^2) = \frac{dy}{dx} = \frac{1}{\sqrt{1-9x^4}} \cdot 6x$

d) $y = \ln(\ln x^2) = \frac{dy}{dx} = \frac{1}{\ln x^2} \cdot \frac{1}{\cancel{x^2}} \cdot 2\cancel{x} = \frac{2}{x \ln x^2}$

11 $f(x) = \sqrt{x^2+9}$ at $x = -4$

$\frac{df}{dx} = \frac{1}{\cancel{2}}(x^2+9)^{-1/2} \cdot \cancel{2}x = \left.\frac{x}{\sqrt{x^2+9}}\right|_{x=-4} = \frac{-4}{\sqrt{16+9}} = \frac{-4}{\sqrt{25}} = -\frac{4}{5}$

$f(-4) = \sqrt{16+9} = \sqrt{25} = 5 \longrightarrow$

$\frac{y-5}{x+4} = \frac{-4}{5} \Rightarrow y = \frac{-4}{5}(x+4) + 5$

$= \frac{4}{5}x - \frac{16}{5} + \frac{25}{5}$

$\Rightarrow L(x) = -\frac{4}{5}x + \frac{9}{5}$

12 $f(x) = 2x^2 - x^2 + 1 \Rightarrow f'(x) = 2 - 2x$

$x_{n+1} = x_n - \left(\frac{2x_n - x_n^2 + 1}{2 - 2x_n}\right) = \frac{\cancel{2x_n} - 2x_n^2 - \cancel{2x_n} + x_n^2 - 1}{2 - 2x_n} = \frac{-x_n^2 - 1}{2 - 2x_n}$

$x_0 = 2$

$x_1 = \frac{-4-1}{2-2 \cdot 2} = \frac{-5}{-2} = \frac{5}{2}$

$x_2 = \frac{-\frac{25}{4} - 1}{2 - \cancel{2}\left(\frac{5}{\cancel{2}}\right)} = -\frac{29}{4} \cdot \frac{1}{-3} = \frac{29}{12}$