

## Math 14 Practice Problems

These problems were compiled by Ed Schaefer. They should not be interpreted to indicate what will be on my tests in any way, but they can give you some practice working on new problems. Unlike an exam, I have ordered the problems so that, to a large degree, they follow the order of the class. This allows you to stop when you encounter material we haven't covered yet. Some problems have hints. If you need the hints, they are between the questions and the answers.

### Questions:

1.  $R(t) = 2\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j}$  sweeps out a curve. Find the  $(x, y)$ -coordinates of the points where the velocity and acceleration vectors are orthogonal.
2. The points  $(0, 10)$  and  $(3, 8)$  break the ellipse  $4x^2 + y^2 = 100$  into two pieces. Set up an integral that gives the arclength of the shorter piece, but don't evaluate it.
3. Let  $C$  be the curve which is the intersection of  $xz - x^2 + y^2 = 0$  and  $x = 3$ . Write down an integral that expresses the arclength along  $C$  between  $(3, 3, 0)$  and  $(3, 6, -9)$ .
4. a) Write down an  $R(t)$  that sweeps out the curve  $C$  which is the intersection of the surfaces  $y = x^2 + 9$  and  $z = 3$ . b) We place a wire along the curve from  $(2, 13, 3)$  to  $(3, 18, 3)$  whose density at a point on the wire is given by its  $x$ -coordinate. Find the mass of the wire. c) Let  $F(x, y, z) = 2y\mathbf{i} - 3z\mathbf{j} + x\mathbf{k}$  be a vector force field. A particle moves along  $C$  from  $(-1, 10, 3)$  to  $(2, 13, 3)$ . Find the work done by the field on it. d) Let  $G(x, y, z) = 2xy\mathbf{i} + (x^2 + 2z)\mathbf{j} + 2y\mathbf{k}$  be a vector force field. Find the work done on the particle in c) but this time without using an integral. (Hint 1)
5. Evaluate the integral  $\int_1^2 \int_{2-x}^x 2xy \, dy \, dx$ . Then sketch the area of the plane that we are integrating over.
6. Compute (Hint: switch)
$$\int_0^{\pi/2} \int_y^{\pi/2} \frac{\cos(x)}{x} \, dx \, dy.$$
7. Compute the average value of  $x + y^2$  over the region bounded by  $y = x$ ,  $x = 3$  and  $y = 0$  in the  $xy$ -plane.
8. a) Write down a formula for the distance between the point  $(x, y)$  and the point  $(0, 0)$ .  
b) Find the average distance to the origin of the points in the unit disk. Reworded: Find the average value of the function in part a) over the region  $x^2 + y^2 \leq 1$ .
9. Graph  $r = \frac{1}{2} - \sin(\theta)$ .
10. Find an  $xy$ -equation for the graph of  $r = 2\cos(\theta)$ .
11. Graph  $\rho = 1 - \cos(\phi)$  in  $xyz$ -space.

12. a) Use spherical coordinates to describe the region above the plane  $z = 1$  and inside the sphere  $x^2 + y^2 + z^2 = 2$ . Your answer should look like  $? \leq \rho \leq ?$ . Hint,  $z = \rho \cos(\phi)$ . b) At what  $\phi$  do these surfaces intersect?
13. Find the volume of the region bounded below by  $z = 0$ , above by  $z = x$  and on the side by  $x^2 + 4y^2 = 4$ .
14. Find the centroid of the region below  $z = x$ ,  $z = -x$ ,  $z = y$  and  $z = -y$  and above  $z = -1$ .
15. Find the volume bounded above by  $x^2 + y^2 + z^2 = 2$  and below by  $z = x^2 + y^2$ .
16. Find the volume in  $\rho = 1 + \cos(\phi)$  and outside  $\rho = 1$ .
17. Find the centroid of the part of the ball  $x^2 + y^2 + z^2 \leq 4$  in the first octant. Hints: find the  $z$ -coordinate first and  $z = \rho \cos(\phi)$ .
18. You hike up a spiral staircase following the path  $R(t) = 3\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j} + 5t\mathbf{k}$ ,  $0 \leq t \leq 10$ . At the point  $(x, y, z)$  your heartrate is  $e^{2z}$ . What was your average heartrate along the path?
19. Find the flux of  $F(x, y) = 2x\mathbf{i} + 3y\mathbf{j}$  across  $r = 2 - \cos(\theta)$ . Hint:  $\int \cos^2(\theta) = \frac{\theta}{2} + \frac{\sin(2\theta)}{4}$ .
20. Find the circulation of  $F(x, y) = y\mathbf{i} - \mathbf{j}$  along the path from  $(0, 0)$  to  $(1, 0)$  to  $(1, 1)$  to  $(2, 1)$  to  $(2, 2)$  to  $(0, 2)$  to  $(0, 0)$ .
21. Find the flux of  $F = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$  across the boundary of  $-2 \leq x \leq 2$ ,  $-3 \leq y \leq 3$ ,  $-1 \leq z \leq 1$ .

### Hints:

1.  $G = \nabla f$ .

### Answers:

1.  $(\pm 2, 0), (0, \pm 3)$
2.  $\int_{\cos^{-1}(\frac{3}{5})}^{\frac{\pi}{2}} \sqrt{25\sin^2(t) + 100\cos^2(t)} dt$
3.  $\int_3^6 \sqrt{1 + (-\frac{2}{3}t)^2} dt$
4. a)  $R(t) = t\mathbf{i} + (t^2 + 9)\mathbf{j} + 3\mathbf{k}$  b)  $\frac{1}{12}(37^{3/2} - 17^{3/2})$  c) 33 d) 60

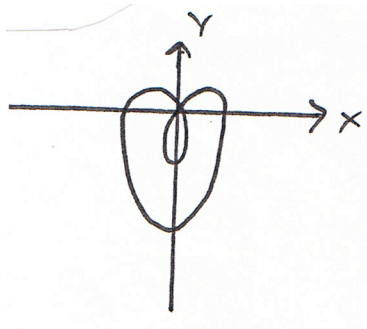
5.  $10/3$

6.  $1$

7.  $7/2$

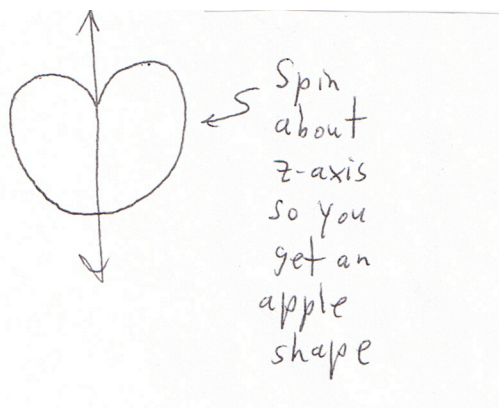
8. a)  $\sqrt{x^2 + y^2}$  b)  $2/3$

9. The graph is



10.  $x^2 + y^2 = 2x$ .

11. The graph has an apple shape as follows:



12. a)  $\sec(\phi) \leq \rho \leq \sqrt{2}$ , b)  $\phi = \pi/4$ .

13.  $8/3$

14.  $(0, 0, -3/4)$

15.  $(-7/12 + 2\sqrt{2}/3)2\pi$

16.  $11\pi/6$

17.  $(3/4, 3/4, 3/4)$

18.  $(e^{100} - 1)/100$

19.  $45\pi/2$

20.  $-3$

21. 288.