Math 11 Practice Problems

These problems were compiled by Ed Schaefer. They should not be interpreted to indicate what will be on my tests in any way, but they can give you some practice working on new problems. Unlike an exam, I have ordered the problems so that, to a large degree, they follow the order of the class. This allows you to stop when you encounter material we haven't covered yet. Some problems have hints.

Questions:

1. If $6\ln(3+x) = 7$ then find x.
2. $\frac{d}{dx}\frac{e^{3x}\sqrt{2-x}}{x^2}$
3. $\frac{d}{dx}(\sqrt{x})^x$
4. $\frac{d}{dx}\ln(\ln(x))$
5. Find the following limits

$$a)\ \lim_{x\to 1}\frac{x^3-x^2-x+1}{x^2-2x+1} \quad b)\ \lim_{x\to 0}\frac{\sin(x^2)}{x^2} \quad c)\ \lim_{x\to -2}\frac{x^2+4x+4}{x^2+3x+2} \quad d)\ \lim_{x\to 4}\frac{\sqrt{8-x}-2}{x-4}$$

$$e)\ \lim_{x\to 2^-}\frac{1}{(x-2)^2} \quad f)\ \lim_{x\to 2^-}\frac{1}{x-2} \quad g)\ \lim_{x\to 2^+}\frac{1}{x-2} \quad h)\ \lim_{x\to 0}\frac{x^2}{x^2+7}$$

$$i)\ \lim_{x\to\infty}\frac{-x^2+1}{x+2} \quad j)\ \lim_{x\to\infty}\frac{-x^2+1}{x^2+2} \quad k)\ \lim_{x\to\infty}\frac{-x^2+1}{x^3+2} \quad l)\ \lim_{x\to\infty}\sqrt{x^2+x}-x$$

6. a) What is the domain of $f(x) = 1/(x-2)$? b) Use the limit definition to find $f'(x)$.
7. a) What is the domain of $g(x) = \sqrt{5-2x}$? b) Use the limit definition to find $g'(x)$.
8. Consider the function

$$f(x) = \begin{cases} x^2 & x < 3 \\ 9 & x = 3 \\ 2x+3 & x > 3 \end{cases}$$

   (a) Does $\lim_{x\to 3} f(x)$ exist?

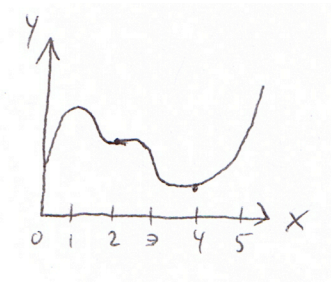
   (b) Is $f(x)$ continuous at $x = 3$?

   (c) Is $f(x)$ diff'able at $x = 3$?

   (d) Find the equations of the tangent lines to the graph of $y = f(x)$ at $x = 1$ and $x = 4$.

   (e) Do those two tangent lines meet? If so where?

9. Make a rough sketch of the derivative of the function graphed below



Hint, where is the slope 0? negative? postive?

10. At what points on the graph of $y = x^2$ does the tangent line pass through $(3, -7)$?

11. a) Find the equation of the tangent line to $y = 3x^2 + 2x + 1$ at $x = -1$. b) Where does this line meet the x-axis? c) Where does this line meet the y-axis?

12. Let x be the number of days since you opened your checking account and let $\frac{-2}{5}x^2 + 40x + 1000$ be the number of dollars in your account on the xth day. a) How much money is in your account on the 10th day? b) What is the derivative on the 10th day? c) Without plugging 11 into anything, use parts a) and b) to estimate how much money is in your account on the 11th day. d) What is the derivative on the 100th day? e) Until which day is the derivative positive? f) On which days is money going into your account faster than it is going out? g) On which day will the amount in your account be the same again as when you opened it?

13. Compute the following derivatives:

$$a)\ \frac{d}{dx}\sqrt{1-x^2} \quad b)\ \frac{d}{dt}\frac{1}{\sqrt{1-t^2}} \quad c)\ \frac{d}{dr}\sqrt{\sin(2r)} \quad d)\ \frac{d}{du}\sin(\sqrt{2u})$$

$$e)\ \frac{d}{dz}\sin(\sqrt{2}z) \quad f)\ \frac{d}{dv}\sin(2\sqrt{v}) \quad g)\ \frac{d}{dx}\sin(\cos(5x+1)) \quad h)\ \frac{d}{dx}\frac{x^2+1}{\tan(x)}$$

$$i)\ \frac{d}{dx}x^2\sqrt[5]{1-x^3} \quad j)\ \frac{d}{dx}\frac{\sec(7x^2)}{6x} \quad k)\ \frac{d}{dx}\cos^3(x) \quad l)\ \frac{d}{dx}3\csc(-7x)$$

14. Find the tangent line to $x = \cos(y/4)$ at $y = \pi$. (Need implicit dif'n).

15. a) $x^2y^2 = 1$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of x and y. b) Evaluate both at $(2, 1/2)$.

16. Let $4x\sin(y) - x^2 = 1$. Find dy/dx where $y = \pi/6$.

17. At what points on the graph of $y^2 + 2y = 4x^3 - 16x - 1$ is the tangent line vertical?

18. a) The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$. You crush a cylindrical can so that its volume stays constant. Find the rate of change of height with respect to the radius. b) Continuing on, say the radius increases at a rate of 3 cm/sec at the moment that $r = 5$ and $h = 4$. What is the rate of change of height with respect to time then?

19. A leak from an oil rig is creating a circular oil slick. Its area increases at a constant rate of 10000π m^2/hour. After 4 hours what is the rate at which the radius is increasing with respect to time?

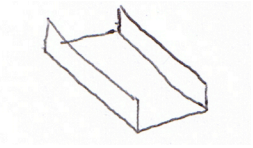
20. Use the principle of the differential to approximate $\sqrt{63}$.

21. Find the linearization to $y = \tan(x)$ at $x = 0$. Do it again at $x = \pi/4$. Pick the best one to approximate $\tan(3\pi/16)$.

22. Find the absolute maximal and minimal values of $f(x) = x^3 - 6x^2 + 9x + 4$ on $[-1, 5]$.

23. Which point on the graph of $x^2 - 2x = 15 - y^4$ has the largest y-coordinate?

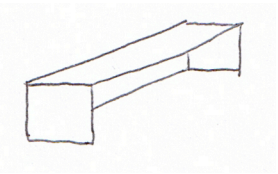
24. A long piece of sheet metal which is 8 cm wide and 1000 cm long is to be made into a horizontal gutter by turning up equal widths along the edges.



How many cm should be turned up on each side to maximize carrying capacity?

25. You make a certain large product. If you sell x of them in a day, your profit is $-x^2+240x-100$ dollars. Due to environmental regulations, the government only allows you to sell at most 100 per day. How many should you sell per day so as to maximize your profit?

26. You build a shelter for the desert with a rectangular top and back and 2 square sides but no front or base. If it had all 6 sides it would be a rectangular box.



You can only afford 384 ft^2. What dimensions give maximal volume?

Answers:

1. $-3 + e^{7/6}$
2. $\frac{e^{3x}\sqrt{2-x}}{x^2}(3 - \frac{1}{4-2x} - \frac{2}{x})$
3. $\frac{1}{2}(\ln(x) + 1)(\sqrt{x})^x$
4. $\frac{1}{x\ln(x)}$

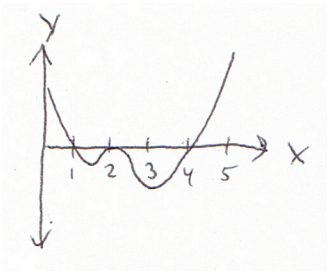
5. a) 2, b) 1, c) 0, d) $-\frac{1}{4}$, e)$+\infty$, f)$-\infty$, g)$+\infty$, h) 0, i)$-\infty$, j) -1, k) 0, l)1/2

6. $x \neq 2, -1/(x-2)^2$

7. $x \leq 5/2, -1/\sqrt{5-2x}$

8. $a)$ yes, $b)$ yes, $c)$ no, $d)$ $y = 2x-1, y = 2x+3$ $e)$ no, parallel

9. The graph looks like



10. $(7, 49), (-1, 1)$

11. $y = -4x-2, x = -1/2, y = -2$

12. $a)$ $1360, b)$ $32, c)$ $1392, d)$ $-40, e)$ $50, f)$ up to $50, g)$ 100. Note that e & f are essentially the same question.

13. $a) -x(1-x^2)^{-1/2}$ $b) t(1-t^2)^{-3/2}$ $c) \cos(2r)/\sqrt{\sin(2r)}$ $d) \cos(\sqrt{2u})/\sqrt{2u}$ $e) \sqrt{2}\cos(\sqrt{2}z)$ $f) \cos(2\sqrt{v})/\sqrt{v}$ $g) -5\sin(5x+1)\cos(\cos(5x+1))$ $h) [2x\tan x - (x^2+1)\sec^2 x]/\tan^2 x$ $i) 2x(1-x^3)^{1/5} - \frac{3}{5}x^4(1-x^3)^{-4/5}$ $j) [14x^2\sec(7x^2)\tan(7x^2) - \sec(7x^2)]/6x^2$ $k) -3\cos^2 x \sin x$ $l) 21\csc(-7x)\cot(-7x)$

14. $y - \pi = -4\sqrt{2}(x - \frac{1}{\sqrt{2}})$

15. $a)$ $-y/x,\ 2y/x^2$ $b)$ $-1/4,\ 1/4$

16. If $y = \pi/6,$ then $x = 1$. Use implicit differentiation : $\frac{dy}{dx} = 0$.

17. $(0,-1),\ (-2,-1),\ (2,-1)$

18. $a)$ $\frac{dh}{dr} = -2V/(\pi r^3)$ $b)$ $\frac{dh}{dt} = -24/5$

19. 25 m/hour

20. 7.9375

21. $y = x,\ y = 1 + 2(x - \frac{\pi}{4}),\ 1 - \frac{\pi}{8}$

22. Max value 24, min value -12.

23. (1,2)

24. 2 cm.

25. 100

26. $8 \times 8 \times 16$