Math 122 Practice Problems

These problems were compiled by Ed Schaefer. They should not be interpreted to indicate what will be on my tests in any way, but they can give you some practice working on new problems. Unlike an exam, I have ordered the problems so that, to a large degree, they follow the order of the class. This allows you to stop when you encounter material we haven't covered yet.

Set #1

1. For the California lottery, you pick 5 different numbers from 1 to 47 and one Mega number from 1 to 27. The Mega number may be the same as one of the five numbers from 1 to 47. What is the probability that you get 3 of 5, but not the Mega number?

2. Stores A, B and C have 50, 75 and 100 employees and, respectively, 50, 60 and 70 percent of these are women. Resignations are equally likely among all employees, regardless of gender. One employee resigns, and this is a woman. What is the probabilty that she works in store C?

3. There are 3 cards. One is red on both sides. One is black on both sides. One is black on one side and red on the other. You can see one card and it is red on top. What is the probability that it is black on the other side?

4. Consider a roulette wheel consisting of 38 numbers: 1 through 36, 0 and double 0. If Eve always bets that the outcome will be one of the numbers 1 through 12, what is the probability that her first win will be on the fourth bet? (She bets all the numbers 1-12 every time).

5. There is a father-son picnic. Jake has two kids. What is the probability that both are boys given that he was invited?

6. There are 4 freshman men, 6 freshman women, and 6 sophomore men. How many sophomore women must be present if the events freshman and woman are independent?

7. An urn contains 5 white and 10 black balls. A die is rolled and that number of balls are chosen from the urn. a) What is the probability that all of the balls selected are white? b) What is the probability that the die landed on 3 if all the balls selected are white?

8. There are five hotels in town. If 3 men check into hotels, what is the probability that they each check into a different hotel?

9. A red die and a blue die are thrown. Let E be the event that the sum of the two dice is 6. Let F be the event that the red die is 2. Are E and F independent?

Answers to set #1

1. $(\binom{5}{3}\cdot\binom{42}{2}\cdot 26)/(\binom{47}{5}\cdot 27) \approx 1/185$. 2. Bayes' $P(W|C)P(C)/(P(W|C)P(C)+P(W|A)P(A)+P(W|B)P(B)) = (.7)(100/225)/((.7)(100/225) + (.5)(50/225) + (.6)(75/225))$. 3. 1/3. 4. $(26/38)^3(12/38)$. 5. 1/3. 6. 9. 7a) $\alpha = \sum_{i=1}^{5} \frac{1}{6}\cdot\binom{5}{i}/\binom{15}{i}$, b) $\frac{1}{6}\binom{5}{3}/(\binom{15}{3}\alpha)$. 8. $5\cdot 4\cdot 3/5^3$. 9. No.

Set #2

1. Y is a discrete random variable with $P(Y=y)=1/(y!e)$ for $y=0,1,2,\ldots$ and $P(Y=y)=0$, otherwise. Find the moment generating function of Y. Use it to find the mean and variance of Y.

2. Let X have probability distribution

$$f(x)=\begin{cases} ke^{-5x} & x\geq 0 \\ 0 & x<0 \end{cases}$$

a) Find the constant k.
b) Find the moment generating function for X. For which t does it exist?
c) Use it to find the mean and variance for X.
d) Find the mean and variance of X directly without using the m.g.f.

3. You toss a fair coin one million times. Use the fact that the normal distribution approximates the binomial distribution for large n and $p=1/2$ to approximate the probability that you get between 499500 and 501000 heads. Hint: Var$=npq$.

4. The mass of silicon chips produced is normally distributed with $\mu=10$ grams and $\sigma=.2$ grams. The cost for making a chip is $1.1+.3\times$(grams of mass) (a \$1.10 production cost, plus 30 cents a gram for silicon). Find the variance of cost.

5. A continuous random variable Y ($Y\geq 0$) has moment generating function $32/(2-t)^5$. Find an interval in which at least 75 percent of the values of Y must exist.

6. The numbers b and c are chosen randomly over the square $0\leq b\leq 1$, $0\leq c\leq 1$. Find the probability that the roots of x^2+bx+c are real.

7. The joint probability function of X and Y is given by

$$f(x,y)=\begin{cases} k(x^2+\frac{xy}{2}) & 0\leq x\leq 1, 0\leq y\leq 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Find k. b) Find the marginal density function of Y. c) Find $P(X>Y)$. d) Find $P(Y>1\,|\,X<\frac{1}{2})$. e) Find $P(X>\frac{1}{2}\,|Y=1)$. f) Find $E(Y)$.

9. On a multiple choice exam with 3 possible answers for each of the 8 questions, what is the probability that a student would get 6 or fewer correct answers just by guessing?

10. People enter a casino at a rate of 1 every two minutes. What is the probability that no one enters between 12:00 and 12:05?

11. You roll a single die. Let Y give the number on top. Find Var(Y).

Answers to set #2

1. $m(t)=e^{e^t-1}$. $\mu=1$, var $=1$. 2. a) 5, b) $m(t)=5/(5-t), |t|<5$, cd) $1/5, 1/25$
3. .8185 4. .0036 5. $|Y-5/2|<\sqrt{5}$ 6. 1/12 7. *a*) 6/7 *b*) $(3y+4)/14$ *c*) 15/56 *d*) 13/20 *e*) 23/28 *f*) 8/7
9. $1-\binom{8}{8}(1/3)^8(2/3)^0-\binom{8}{7}(1/3)^7(2/3)^1$ 10. $e^{-2.5}(2.5)^0/0!=e^{-2.5}$
11. $\frac{1}{6}(1+4+9+16+25+36)-(3.5)^2$

Set #3

1. The distribution of the length of the tail of deer in the hills has $\mu_h = 10$ and $\sigma_h = 2$ centimeters. For those in the valley it is $\mu_v = 10.5$ and $\sigma_v = 2.4$. Find n so that if you took a sample of n deer from the hills and n deer from the valley, then the sample mean for valley deer would be larger than the sample mean for hill deer with probability 90%.

2. 5% of people are biologically predisposed to alcoholism. SCU has 4400 undergraduates. Approximate the probability that fewer than 200 are so predisposed. (Assume that SCU students form a random sample, also in problem 3).

3. A study of the life span of portable radios found the average to be 3.1 years with a standard deviation of 1.5 years. If the number of such radios owned by students in a dorm is 225, approximate the probability that the mean lifetime of these radios will be less than 3 years.

4. I take a sample of 100 SCU students' IQ's to approximate the actual mean of SCU IQ's. I want to claim that I am off by at most 1 with certainty 95%. How much variance is allowed in IQ's?

5a) Let X and Y have joint pdf given by $x + y$ on $0 \le x \le 1$, $0 \le y \le 1$ and 0 elsewhere. Find the coefficient of correlation for X and Y. (Using symmetry, this can be done with 3 double integrals.)
b) Find $\mathrm{Var}(3X + 5Y)$ (this should take no new integrals).

6. X and Y each take on values 0 and 1 only and are independent. In the margins are written the marginal probabilities. Fill in the joint probability distribution.

		X = 0	X = 1	
Y	0			1/4
	1			3/4
		1/3	2/3	

7. The joint pdf of X and Y is below. Are they independent?

$$f(x,y) = \begin{cases} \frac{6}{7}(x^2 + \frac{xy}{2}) & 0 \le x \le 1, 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

8. Eight ducks fly over 10 hunters. The hunters choose targets randomly and independently. They each fire one shot simultaneously. Each hunter has a 0.6 chance of hitting his target. What is the expected number of ducks that will get hit? Hint: let $X_i = 1$ if the ith duck is shot and $X_i = 0$ if not.

9. Let X and Y be independent random variables with pdfs $f(x) = e^{-x}$ for $x \ge 0$ and $f(y) = e^{-y}$ for $y \ge 0$. Find the pdf for $U = X + Y$.

Answers to set #3

1. 64 2. .0838 3. .1587 4. $(10/1.96)^2$ 5a) $-1/11$ b) 43/18
6. Top row: $1/12, 1/6$. Bottom row: $1/4, 1/2$. 7. No $f_X(x) = (12x^2 + 6x)/7$, $f_Y(y) = (3y + 4)/14$, $f_X(x)f_Y(y) \ne f(x,y)$. 8. $8(1 - (\frac{7}{8} + \frac{1}{8}(0.4))^{10})$. 9. $f(u) = ue^{-u}$ for $u \ge 0$.