

Axioms:

Where $\vec{u}, \vec{v}, \vec{w} \in V$, and $k, m \in \mathbb{R}$

1. $\vec{u} \oplus \vec{v} \in V$ | Closure Under Addition
2. $\vec{u} \oplus \vec{v} = \vec{v} \oplus \vec{u}$ | Commutative Property
3. $\vec{u} \oplus (\vec{v} \oplus \vec{w}) = (\vec{u} \oplus \vec{v}) \oplus \vec{w}$ | Associative Property of Vector Addition
4. $\forall \vec{u} \in V \exists \tilde{\mathbf{0}} \in V$ s.t. $\vec{u} \oplus \tilde{\mathbf{0}} = \tilde{\mathbf{0}} \oplus \vec{u} = \vec{u}$ | Additive Identity
5. $\forall \vec{u} \in V \exists (-\vec{u}) \in V$ s.t. $\vec{u} \oplus (-\vec{u}) = (-\vec{u}) \oplus \vec{u} = \tilde{\mathbf{0}}$ | Additive Inverse
6. $k \odot \vec{u} \in V$ | Closure Under Scalar Multiplication
7. $k \odot (\vec{u} \oplus \vec{v}) = k \odot \vec{u} \oplus k \odot \vec{v}$ | Distributive Property Of Scalars
8. $(k + m) \odot \vec{u} = k \odot \vec{u} \oplus m \odot \vec{u}$ | Distributive Property On Vectors
9. $k \odot (m \odot \vec{u}) = (k \cdot m) \odot \vec{u}$ | Associative Property of Scalar Multiplication
10. $1 \odot \vec{u} = \vec{u}$ | Multiplicative Identity

Theorems:

Where $\vec{u} \in V$ and $k \in \mathbb{R}$

1. $0 \odot \vec{u} = \tilde{\mathbf{0}}$
2. $k \odot \tilde{\mathbf{0}} = \tilde{\mathbf{0}}$
3. $(-1) \odot \vec{u} = (-\vec{u})$
4. $k = 0$ XOR $\vec{u} = \tilde{\mathbf{0}}$ IF $k \odot \vec{u}$